FIVE-AXIS MILLING OF SPHERICAL SURFACES

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Abstract—A new five-axis technique for machining of spherical surfaces with a radiused or flat bottom end mill is presented. The tool path generated by the proposed technique produces scallop free surfaces at a fraction of the time required by conventional milling using a ball nose end mill. The proposed method was verified experimentally on a wrist type and on a tilt-rotary table type five-axis milling machine.

1. INTRODUCTION

Complex surfaces are encountered in many objects such as turbine blades, automobile parts, airplane components, injection moulds and dies. The 3D description of these surfaces is often produced by combining together elements such as: cylinders, cones, spheres, Bezier, and spline surfaces using a CAD system [1]. This geometric description can then, for example, be used to generate a tool path for controlling a milling machine.

The actual machining of a complex surface can be broken down into three main steps. During the initial roughing cuts, a large amount of material is removed to sculpt the general shape of the surface, as quickly as possible, without bringing the tool in contact with the desired surface. After roughing, finishing cuts are used to remove the remaining material. Typically a single point on a ball nose end mill is used to generate the desired surface. This operation results in a surface with a large number of scallops as shown in Fig. 1. Grinding and polishing, called benchwork, is then used to remove these scallops. A recent survey by LeBlond Makino of Mason, Ohio [2], stated that even a small mould will typically require 57 hr of roughing, 127 hr of finishing, and 86 hr of benchwork. Almost 80% of the total machining time is spent on finishing and benchwork. Clearly, there is a need for faster machining techniques which produce smaller scallops, and hence require little or ultimately no benchwork.

![Diagram of scallops](image)

Fig. 1. Scallops left behind when machining with a ball nose end mill.

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The problems associated with the current practice of milling complex surfaces with ball nose end mills has led to promising research into five-axis machining. Vickers et al. [3] described a five-axis method of machining compound curvature surfaces with a flat bottom end mill. In their method the tool is inclined in the direction of feed which allows the programmer to select an effective cutting radius for a given surface. Jensen et al. [4] extended this work by changing the inclination angle of the tool at every point on the surface. The cutter's swept silhouette could now be matched to the surface topology more closely. Independent work by Jensen et al. [5] and Bedi et al. [6] increased the flexibility of the technique by extending the concept of curvature matching to the radiused end mill.

All previous work on five-axis milling of complex surfaces using flat or radiused end mills has focused on the idea of matching the curvature of a point on the tool as closely as possible to the curvature of a point on an arbitrary surface. These techniques can be applied to any surface subject to interference limitations, but may not be the best technique for a particular surface. This paper will focus on the commonly occurring but difficult to machine spherical surface. Such a surface is often used in joints, moulds, valves, bearings, etc. Spherical surfaces are easily produced on a lathe. However, if the workpiece is large and complex, or the spherical surface is not centred on the workpiece, it may be more desirable to produce the surface on a milling machine.

In conventional three-axis machining of spherical surfaces, a ball nose cutter would be used for finishing as shown in Fig. 1. The tool is positioned by bringing a point on the cutter in contact with a point on the desired surface such that both points share a tangent plane. The tool path is generated by moving the cutter between a number of these points using linear or circular interpolation. The distance between adjacent sections of the tool path is called the step-over.

The proposed technique, may best be explained by what the authors call the "drop the coin" concept. If you place a coin in a spherical dish, every point on the coin's circumference will contact the surface. In geometric terms, a plane will intersect a sphere in a circle. As the coin moves around the surface, every point on the coin's circumference remains in contact with the surface. The distance between the centre of the sphere and the centre of the coin is a constant which can be calculated. In the proposed technique, the coin is replaced by the flat bottom of an end mill. The concept can be extended to radiused end mills by identifying circles of tangency between the radiused end mill and the spherical surface. In this paper, the theory is developed for the general case of the radiused end mill. A flat end mill is considered to be a special case of a radiused end mill with a corner radius of zero.

The present work compares the conventional three-axis method of machining the interior and exterior spherical surfaces with the proposed five-axis technique. In sections 2 and 3 relationships between scallop height, path length, and tool geometry will be developed for both approaches. The implementation of the new technique will be discussed in section 4, for a wrist type five-axis milling machine. Sections 5 and 6 contain a comparison of both approaches and the experimental verification obtained on a wrist type and on a tilt-rotary table type five-axis milling machine.

2. CONVENTIONAL MACHINING TECHNIQUE

2.1. Spherical dish

Throughout this paper, the interior of a spherical surface will be simply referred to as a dish. The reference frame will be placed at the centre of the dish. The bottom of the dish will lie on the negative Z-axis and all angles will be measured from the positive X-axis. Figure 2 shows a cross-section of the dish in a plane passing through its centre and two adjacent tool positions. The tool positions, $P_i$ and $P_{i+1}$, are separated by an angular step-over, $\beta$. The axis of the tool is along the Z direction. The dish has a radius of $R$ and the cutter has a radius of $r$. Since the curvature of the cutter and the desired surface do not match, a scallop with a height of $h$ is left between adjacent tool positions. For the condition of tangency to be satisfied, the radius of the dish at
points $P_i$ must pass through the centre of the ball nosed cutter at $C_i$. The distance between the centre of the dish and the centre of the tool, referred to as the tool space radius, $R_{ts}$, is a constant for all points and is equal to:

$$R_{ts} = R - r.$$  

Figure 3 shows the two adjacent segments of the tool path corresponding to $P_i$ and $P_{i+1}$ in Fig. 2. The tool centre, $C_i$, moves in an arc from the lip of the dish to its bottom on the spherical surface defined by the tool space radius. Using the geometry in Fig. 2, the scallop height, $h$, can be expressed by equation (1). The maximum scallop height occurs at the lip of the dish when $\beta'$ is a maximum and decreases to zero as $\beta'$ decreases to zero at the bottom of the dish, where the angle $\beta'$ is related to $\beta$ according to the equation $\beta' = \beta \sin(\theta)$, see Fig. 3.

$$h = R - (R - r) \cos\left(\frac{\beta'}{2}\right) \sqrt{\left(r^2 - (R - r)^2 \sin^2\left(\frac{\beta'}{2}\right)\right)}.$$  

(1)

Rearranging equation (1) and solving for $\beta'$ yields:

Fig. 3. Tool path of a ball nose end mill machining a dish.
\[
\beta' = 2 \cos^{-1}\left(\frac{h^2 - 2hR - 2rR + 2R^2}{2(R^2 - Rr - Rh + rh)}\right).
\] (2)

The path length and thus the machining time can be estimated for a given scallop height at the lip of the dish using the equation:

\[
P = \frac{2\pi R_{\text{cu}}}{\beta}
\] (3)

where \(\alpha\) is half the cone angle subtended by the dish.

2.2. Spherical dome

A dome can be machined on a three-axis milling machine using a ball nose end mill in a manner similar to that used for a dish. The reference frame will be placed at the centre of the dome, the top of the dome will lie on the positive Z-axis and all angles will be measured from the positive X-axis. Figure 4 shows a cross-section in a plane containing the centre of the dome and two adjacent tool positions, \(P_i\) and \(P_{i+1}\), which are separated by the angular step-over \(\beta\). In this figure the tool axis is along the Z direction. The tool space radius, \(R_{\text{cu}}\), for the dome is the sum of the dish radius, \(R\), and the radius of the cutter, \(r\), i.e.

\[
R_{\text{cu}} = R + r.
\]

Using the geometry shown in Fig. 4, the scallop height, \(h\), and angular cross over, \(\beta'\), are given below

\[
h = (R + r) \cos\left(\frac{\beta'}{2}\right) - R - \sqrt{(r^2 - (R + r)^2 \sin\left(\frac{\beta'}{2}\right))}
\] (4)

\[
\beta' = 2 \cos^{-1}\left(\frac{2R^2 + 2Rh + h^2 + 2Rr}{2(R^2 + Rr + Rh + rh)}\right).
\] (5)

The path length can be obtained from equation (3). In the above derivations, the tool path consisted of circular arcs on the sphere defined by the tool space radius, similar to the longitudinal lines on a globe. This path might not be the most efficient for three-axis machining, nor the one that leads to the smallest scallop height. Nevertheless, it will give us some basis for comparison with our proposed technique described next.

![Fig. 4. Geometry of a ball nose end mill machining a dome; cross-section plane passing through the centre of the dome and two points of tangency.](image)
3. PROPOSED FIVE-AXES TECHNIQUE

3.1. Spherical dish

In the “drop the coin” concept, a coin placed in a spherical dish will contact the surface at every point on its circumference. Figure 5 shows a cross-section through the tool axis of a radiused end mill machining a dish. The points of tangency between the tool and the dish form a continuous circle when the tool axis is collinear with the surface normal. This figure shows that the surface normals at \( T_1 \), \( T_2 \), and \( P \), are collinear with the corner radii and tool axis, respectively. When these conditions are met the centre of the sphere forms an isosceles triangle with points \( T_1 \) and \( T_2 \) which can be used to calculate the correct position of the tool centre. In this paper, the tool centre, \( C \), is defined as the point of intersection of the line joining the two centres of the corner arcs and the tool axis. The proper tool placement can be determined by calculating the tool space radius using the equation below:

\[
R_{ts} = (R - r) \cos \left( \frac{\beta}{2} \right).
\]  

(6)

The diameter of the circle of contact between the dish and the radiused end mill, called the effective diameter, \( D_e \), can be calculated by:

\[
D_e = R \left( \frac{D - 2r}{R - r} \right)
\]  

(7)

where \( D \) is the diameter of the tool. Accordingly, the maximum angular cross step will be:

\[
\beta = 2 \sin^{-1} \left( \frac{D_e}{2R} \right).
\]  

(8)

The tool path in this work consists of a number of concentric circles which are parallel with the \( XY \) plane as shown in Fig. 6. The centre point of the tool moves along each of these circles in such a way that the tool remains collinear with the surface normal. Each circle is inscribed on the sphere defined by the tool space radius. For a constant value of \( \beta \) between adjacent circles, the tool path length can be approximated by summing up the length of each circle as given below.

![Fig. 5. A radiused end mill machining a dish where tangency is a full circle of diameter \( D_e \).](image)
Fig. 6. Tool path for five-axis machining of spherical surface.

\[ P = 2\pi R_0 \sum_{i=1}^{n} \cos(\beta_{\text{start}} + i\beta) \]

\[ n = \text{Integer} \left[ \frac{\alpha}{\beta} \right] + 1 \] (9)

where \( \beta_{\text{start}} \) is the tool angle at the lip and \( \alpha \) is half the cone angle subtended by the dish.

3.2. Spherical dome

A convex spherical surface, called a dome, can be cut using the same five-axis technique described above. However in this case, the technique requires a special type of radiused end mill, called a toroidal cutter. A cross-section of the correct placement of a toroidal cutter machining a spherical dome is shown in Fig. 7. The toroidal cutter used in the experiments consisted of two circular carbide inserts of radius \( r \) placed in a tool holder of diameter \( D \). When the tool is rotating, the inserts produce a donut or a toroidal cutting surface. Provided the core of the donut is not solid, it is possible to cut a spherical surface on the interior portion of the torus. Two conditions must be satisfied for the desired circle of tangency between the tool and the dome. The surface normals at \( T_1 \), \( T_3 \) and \( P \) are collinear with the insert radii and the tool axis, respectively. When these conditions are met the centre of the sphere forms an isosceles triangle.

Fig. 7. A toroidal cutter machining a dome using the “drop the coin” concept.
with the centres of the circular inserts, which can be used to calculate the correct position of the tool. Assuming that the tool has two inserts, the tool centre is defined as the point of intersection of the line joining the two centres of the inserts and the tool axis.

The tool space radius, \( R_{ts} \), and the effective diameter, \( D_e \), will be calculated from:

\[
R_{ts} = (R + r) \cos \left( \frac{\beta}{2} \right) \tag{10}
\]

\[
D_e = R \left( \frac{D - 2r}{R + r} \right). \tag{11}
\]

The expression for the angular step-over, \( \beta \), remains the same as in the case of the dish.

By examining equation (11) a few conclusions can be made. As the separation between the two inserts increases the effective diameter increases. This can be accomplished by increasing the cutter diameter or decreasing the insert diameter. As the separation between the inserts decreases the tool begins to look more like a ball nose cutter and the effective diameter approaches zero. A toroidal cutter cannot machine a dome with a radius less than the tool’s effective radius. However, for very small spheres the clearance between the sphere and the bottom of the tool holder may become the limiting factor. There is no upper limit to the radius of curvature of the spherical surface machinable, i.e. this tool can be used to cut a plane.

As in the case of the dish, the tool path consists of a number of concentric circles which are parallel with the \( XY \) plane. The centre point of the tool moves along each of these circles in such a way that the tool axis remains collinear with the surface normal. The tool path length can be approximated by summing up the length of each circle using equation (8).

4. IMPLEMENTATION

Machining of a spherical dish was implemented on a five-axis wrist type machine with a flat end mill, while the dome was machined on a tilt-rotary table type machine with a toroidal cutter. Since the approach is the same in both cases, only the implementation of the dish machining will be described in detail.

Figure 8 shows the five-axis wrist type milling machine cutting a dish. In this set-up the dish is clamped on the machine table which moves along the \( X \)-axis. The wrist translates along the \( Y \)- and \( Z \)-axes. The spindle motor is mounted at the wrist of the machine and can rotate about the \( A \)- and \( C \)-axes. The \( A \)-, \( C \)- and spindle-axes all intersect at a single point called the wrist centre. The distance between the wrist centre and the centre of the tool is \( L \).

As the tool moves around the dish, the tool axis pivots about the centre of the sphere, which makes the centre of the sphere the logical choice for the work coordinate system (wcs). Once the work coordinate system is established, the position \([X, Y, Z]\) of the wrist centre relative to the work coordinate system, and the orientation of the tool \([A, C]\) must be determined. The bottom of the tool contacts the desired surface on its circumference. Instead of trying to calculate the points of contact between the desired surface and the tool, the centre of the tool is positioned on the sphere defined by the tool space radius, \( R_{ts} \).

The parametric equation of a point \( P_{ts} \) relative to the work coordinate system is given by the equation below in spherical coordinates. The angles \( \theta \) and \( \phi \) are measured from the positive \( X \)-axis as shown in Fig. 6.

\[
\begin{bmatrix}
P_{ts}' \\
P_{ts}' \\
P_{ts}'
\end{bmatrix}
= \begin{bmatrix}
R_{ts} \cos(\theta) \cos(\phi) \\
R_{ts} \cos(\theta) \sin(\phi) \\
R_{ts} \sin(\phi)
\end{bmatrix}. \tag{12}
\]
The position vector of the tool centre, $P_c$, relative to the wrist centre is given by equation (13).

$$
\begin{bmatrix}
P_{c x} \\
P_{c y} \\
P_{c z}
\end{bmatrix} =
\begin{bmatrix}
L \cos(A) \cos(C) \\
L \cos(A) \sin(C) \\
-L \sin(A)
\end{bmatrix}.
$$

Provided that the $A$- and $C$-axes initially have the same orientation as the $Y$- and $Z$-axes of the work coordinate system, the tool can be oriented by ensuring that the values of $A$ and $C$ are the same as $\theta$ and $\phi$. The position of the wrist centre relative to the work coordinate system is then given by:

$$P_{wc} = P_{ts} - P_c.$$

The path is produced by moving the tool using linear interpolation between the set of points, each with coordinates $[P_{wc}, A, C]_m$. Care must be taken to use small incremental angles to ensure that the path between points is reasonably linear.

Programming the tilt-rotary table type machine is slightly more complicated. The centre of the dome is still the logical choice for the work coordinate system, and the tool axis is still collinear with the surface normal, and passes through the centre of the sphere. However, the tool axis no longer passes through the intersection of the table's rotary axes. In addition, movement of the rotary axes changes the orientation and position of the work coordinate system. The solution to this problem is to use a homogeneous transformation matrix to transform the work coordinate system to global coordinates for a given workpiece orientation. The work by Rao et al. [7] contains a complete development of the kinematics of the five-axis tilt-rotary type milling machines used for this study.

5. NUMERICAL EVALUATION

The two important points of comparison between the traditional and the new approach are the amount of time required to machine the surface and the resulting surface finish. When using the conventional three-axis approach to the machining of
spherical surfaces the scallops depend on the step-over. The benchwork required to remove these scallops is time consuming. Scallop size can be reduced by using a smaller step-over but this solution results in significantly greater machining times. The manufacturing engineer is faced with the problem of finding the optimum amount of time to spend on machining vs grinding and manual polishing. In addition, he or she will have to be concerned with the loss of dimensional accuracy inherent in manual work.

In the proposed approach scallops are virtually eliminated. There is little advantage in terms of surface finish of having a step-over smaller than the effective diameter. The surface finish is dependent on only the cutting conditions, machine rigidity, and the distance between points used for linear interpolation. These factors are always present when machining 3D surfaces in addition to the geometrical aspects of scallop generation. Time savings associated with the proposed technique are realized due to the increase in step-over and the elimination of manual removal of scallops. In addition, there is no loss of dimensional accuracy due to manual work.

Table 1 compares the maximum step-over and the corresponding path length required by the two techniques when milling a 20 mm deep dish with a 50 mm radius for a maximum scallop height of 0.01 mm. For simplicity, a regular end mill is used for the proposed technique. Values were calculated using the equations derived in sections 2 and 3. The table clearly shows the speed advantage of the new technique over the conventional method. For the same diameter tool, a larger step-over can be used which results in approximately a magnitude reduction in tool path length and machining time.

Table 2 compares the step-over and path length obtained when machining a dome. A 0.01 mm scallop height was used in the calculation for the ball nose end mill. The inserts on the toroidal cutter were 9.525 mm (3/8") diameter. For the same speeds and feed rates, the new technique should produce a spherical surface with a much better surface finish more than 10 times faster than the old technique. This table also demonstrates the effect of insert size on effective diameter. The effective diameter of the second tool was 44% larger than the effective diameter of the first one while the actual diameter was only 25% larger. This difference occurs because the same size inserts were used in a larger tool holder. For fast machining, large toroidal cutters with small inserts should be used provided there is no interference with the bottom of the tool holder.

### Table 1. Comparison between the proposed and conventional methods of machining a spherical dish

<table>
<thead>
<tr>
<th>Tool radius (mm)</th>
<th>Step-over (mm)</th>
<th>Path length ( (PL_1) ) (mm)</th>
<th>Step-over (mm)</th>
<th>Path length ( (PL_2) ) (mm)</th>
<th>( PL_1/PL_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.54</td>
<td>18,187</td>
<td>7.98</td>
<td>835</td>
<td>21.8</td>
</tr>
<tr>
<td>8</td>
<td>0.73</td>
<td>11,216</td>
<td>15.86</td>
<td>415</td>
<td>27.0</td>
</tr>
<tr>
<td>12</td>
<td>0.85</td>
<td>7880</td>
<td>23.53</td>
<td>273</td>
<td>28.9</td>
</tr>
</tbody>
</table>

### Table 2. Comparison between the proposed and conventional methods of machining a spherical dome

<table>
<thead>
<tr>
<th>Tool diameter (mm)</th>
<th>Step-over (mm)</th>
<th>Path length ( (PL_1) ) (mm)</th>
<th>Step-over (mm)</th>
<th>Path length ( (PL_2) ) (mm)</th>
<th>Effective diameter (mm)</th>
<th>( PL_1/PL_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.4</td>
<td>1.13</td>
<td>16,238</td>
<td>15.8</td>
<td>554</td>
<td>14.5</td>
<td>29.3</td>
</tr>
<tr>
<td>31.75</td>
<td>1.29</td>
<td>15,641</td>
<td>21.9</td>
<td>380</td>
<td>20.3</td>
<td>41.2</td>
</tr>
</tbody>
</table>
When comparing the three-axis data with the five-axis data the reader should be aware that no attempt was made here to optimize the three-axis tool path. Undoubtedly more efficient tool paths exist but are unlikely to offer the magnitude of improvement achievable by the "drop the coin technique".

6. EXPERIMENTAL VERIFICATION

An aluminium dome and a dish were machined on two different types of five-axis milling machines to validate the proposed technique. Each piece was roughed using a three-axis tool path. Each surface had a radius of 50 mm and a depth of 20 mm. The dish was cut at the Industrial Research and Development Institute (IRD1) in Midland, Canada, on a JO 'TECH five-axis wrist type CNC milling machine (JTH 50). The following cutting conditions were used: spindle speed = 20,000 rpm, feed = 0.0325 mm/tooth, 0.5 mm depth of cut, using a 2 flute 10 mm diameter end mill. A C++ program was used to calculate the tool position and convert the resulting data into the G-code. The resulting tool path consisted of 1260 points. Linear interpolation was used to move between the points. The cutting time was 112 sec. A photograph of the dish and a close up of the surface are shown in Fig. 9. The surface finish of the dish was similar to that of a plane machined by an end mill and was smooth to the touch. The circulate marks on the surface are a result of the change in direction and speed at the end of each linear interpolation and the regular feed marks. For comparison, the same sized dish was produced using a ball nose end mill as shown in Fig. 10. Talyrond measurements, shown in Fig. 11, were done on both spherical cavities at a depth of 1.7 mm from the lip. Both techniques produce reasonably round spherical cavities, but the three-axis technique using a ball nose mill produced a maximum in the plane deviation of approximately 30 μm, whereas the new technique results in a mere 2.5 μm deviation.

The dome was milled on a FADAL VMC 4020 five-axis tilt-rotary table type milling machine at the University of Western Ontario, London, Canada. The dimensions of the sphere were the same as the dish. The cutting conditions used were: spindle speed = 3300 rpm, feed = 0.0325 mm/tooth, 0.5 mm depth of cut, using a 25.4 mm diameter toroidal cutter with 2, 9.525 mm diameter carbide inserts. The tool path consisted of 490 points. Linear interpolation was used to move between the points. A photograph of the finished product and a close up of the surface are shown in Fig. 12. Talyrond measurements were also conducted on the sphere at a level of 9.4 mm from the top of the sphere. The resulting polar graph shown in Fig. 13 confirms that the sphere had good roundness with a maximum radial error of 9 μm.

7. DISCUSSION

A new five-axis technique for the machining of spherical surfaces has been described and tested experimentally. These cutting tests have shown that the error in the dome was four times that of the dish as evidenced by the arcs visible on the close up in Fig. 12. It turned out that there was a significant misalignment (estimated at 0.2°) between the rotary table and the translational axes in the former set-up. This indicates that the proposed method is sensitive to this type of misalignment. This adverse effect, however, could be utilized in evaluating the misalignment, by mounting an accurate sphere and replacing the tool with a suitable probe. This idea is being pursued by the authors to speed up the alignment process of five-axis machines.

Also, in implementing the new technique, the machined parts were deliberately chosen shallow to avoid interference. Obviously, this interference limitation represents the major constraint on the proposed and perhaps on other five-axis machining techniques. At best, we should look at the results obtained here as an upper bound where it was possible to generate the sphere using 1/2 the circumference of the tool. This is a drastic improvement over generating the surface at one point using a ball nose end mill in a three-axis set-up.

At present the authors are investigating a new strategy where general surfaces are
generated at more than one point using flat and radiused end mills. This Multi Point
Machining approach or, simply, MPM, will attempt to minimize the material left behind
subject to constraints of interference and gouging. The technique of machining a dish
and dome presented in this paper is but one specific application of the approach.

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REFERENCES

Fig. 10. Photographs of the dish machined using the three-axis method with a ball nosed end mill.
Fig. 11. Talyrond records of machined dish: (a) five-axis method using regular end mill, (b) three-axis method using ball nose end mill.
Fig. 12. Photographs of the dome machined using the five-axis method with a toroidal cutter.
Fig. 13. Talyrond measurements of a dome machined using five-axis method and toroidal cutter.