Swept volumes of toroidal cutters using generating curves

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Abstract

Modelling the volume swept by a tool as it moves from one programmed position to another in three, four and five-axis milling is a challenging task. In this paper, a technique for generating the volume swept by a toroidal cutter in its motion along a given tool path, is presented. This technique is based on identifying "generating curves" along the path and connecting them into a solid model of the swept volume. The swept volume simulations are verified experimentally for three test pieces. These are also compared with the simulation results from the Z-map technique. The results of the experimental verification show the method to be accurate to within 10 μm for the three test pieces. Furthermore, the computation time for the new technique is significantly less than the Z-map method. It is concluded that the proposed method has the potential of allowing fast simulation and verification of multi-axis tool paths. © 1998 Elsevier Science Ltd. All rights reserved.

1. Introduction

Sculptured surfaces and other geometrical entities in tools, dies and moulds are traditionally machined on three-axis milling machines with ball nosed end milling cutters. These cutters have poor machinability near the bottom and the finish machining time can be large for a reasonable surface finish. Figure 1 shows the cutters commonly used in milling, they are: ball nose, flat bottom and toroidal end mills. Recently, work advocating the use of five-axis machines to reduce machining time (by up to 85%) has become available in the literature and will be at the forefront of research agenda in coming years [1–6]. These five-axis machining methods are based on match-
Fig. 1. Commonly used cutters.

ing the geometric characteristics of the surface with that of a toroidal tool (e.g. in the Principal Axis Method (PAM)) [3,4]; and on machining a surface by utilizing two points of contact between the tool and workpiece (e.g. in the Multi-Point Machining) [6]. One critical issue in using these methods is the production of a gouge and interference free tool path. The key difficulty is the determination of the surfaces generated by the tool as it moves relative to the workpiece. This problem, called "the swept volume" of a tool, is an ongoing research challenge.

Attempts to solve the swept volume problem can be classified into theoretical methods and procedural methods. The theoretical methods include the envelope method [7], the Lie group method and the differential manifold method [8–10]. The swept volume for a moving toroidal tool in all these methods would be generated by combining the volumes swept by a model of the tool shank (cylinder), the tool cutting edges (torus) and the bottom (circle). The individual swept surfaces are generated using different theories. These swept entities must first be realized and then combined by finding intersection curves, into a valid model. Since the swept volumes are represented by complex algebraic/trigonometric equations, finding intersection curves is a computationally heavy task. Furthermore, realizing these volumes into a solid model, (which can represent only a limited number of primitives), may require approximation methods for both topology and geometry. It is these two steps which take up a considerable amount of time and make the above methods impractical. The procedural methods include the sum of volumes [11,12] the intersection of vectors [13], and the ray casting methods [14,15]. The swept volume of an object can be generated by a sum of the volumes of its components. Generating each component is a simple task but a large number of these operations is required which increases the overall time of computation significantly. One alternative of this method is the creation of numerous images of the tool from the start to the end of the pass and combining them to generate the swept volume for the move. Although it is simple to implement, the method is very time consuming. For one of the test tool paths used in the current work, this method was implemented on a Pentium 75 MHz PC. For this tool path with 140 moves (each move is 1 mm) it took 14 hours when the tool was represented with exact surfaces. The time was reduced to 7 hours when the tool was modelled
using faceted models. In the intersection of vectors approach [13], the swept volume is created by intersecting rectangular and circular sections with the surface. The difficulty in this technique is to find a valid search interval of the independent variables to be used in the highly nonlinear and complex differential equations. The ray casting method uses a large number of rays emanating from the part surface and intersects them with the tool at any given position. To generate the swept volume, numerous images of the tool are placed between the starting point and the endpoint. The intersection process is repeated for each of these positions and a good approximation of the swept surface can be generated. This method has been converted to work on parallel processor based systems [14] and gives good performance. However, the cost of the system is high. This method has also been implemented by Rao et al. [15] to generate instantaneous swept volumes for gouge avoidance and determination of cutting forces.

This paper overcomes the speed and memory problems associated with the above methods and presents an approximate method of generating swept volumes for multi-axis tool paths. Moreover, while existing techniques have focused on ball nosed and flat end mills, the present paper addresses the volumes swept by toroidal cutters. Such cutters inherit the merits of both the above cutters, and are gaining wider applications in industry [16].

2. Generating curves

To understand the concept of establishing swept volumes by toroidal cutters using “generating curves”, one must first look at the simpler case of a flat bottom end mill generating a spherical cavity. Warkentin et al. [6] have shown that when the mill rotates about a point on the tool axis it produces part of a sphere. It is the bottom circular end of the cutter that generates the spherical surface; it is called here the “generating circle”. The radius of this circle is the radius of the cutter \( R \), and it remains unchanged regardless of the radius of the spherical cavity. A toroidal cutter rotating about a point on its axis also generates a spherical surface at the bottom. The sphere is also produced by a circle which is centered on the tool axis and lies in a plane perpendicular to the tool axis. This circle is identified by joining the center of rotation and the center of inserts and extending the line to intersect the boundary of the insert as shown in Fig. 2. This is the “generating circle” that generates the spherical surface; it is as if the toroidal cutter has been replaced by a flat end mill. The radius of the generating circle, however, varies according to the curvature of the surface to be machined. In effect, one can approximate the toroidal cutter by numerous flat end mills; at any point along the tool path, only one flat end mill approximates the toroidal cutter as depicted in Fig. 2. While finding the swept volume using the “generating circles” is exact for spherical cuts, as it was verified experimentally by Warkentin et al. [6] it is only an approximate approach for general surfaces. For such surfaces, curvature could vary along as well as across the feed direction. However, for open form surfaces, typically encountered in moulds and dies, it is reasonable to assume that the arc of cut within a short movement of the tool along the feed direction to be circular. In this study, it is assumed that during this short movement, the tool axis remains in the “feed plane” which also includes the feed direction. Across the feed direction, it is not the “generating circle” but rather its projection onto the “osculating plane” that determines the swept volume profile, as shown in Fig. 3. The osculating plane, or the cutting plane, is a plane normal to the feed direction at the point of contact between the tool and workpiece. This
profile matches the intended surface profile exactly at the center of the tool in the feed plane, and deviates from it as we move sideways to other sections.

3. Implementation

Although the concept of using the "generating circles" technique for computing the swept volume is general, as yet we have only managed so far to implement it for cases where the tool path is described in one plane that includes the tool axis. In these cases the surface swept by the circles does not exhibit "twists" and "turns". These twists and turns would necessitate breaking up the tool path into segments, each in a different plane, and a more comprehensive investigation would be needed to assess the effect of the length of these segments on the resulting swept volume. Also, we will assume that the design surface patch under consideration does not include degeneracies or loops.

The computation of the swept volume starts with the given tool path described in terms of G-code or in APT format. In both cases the location of the tool and its orientation can be determined from the information in the code, the parameters of the tool including its length, and the kinematics of the CNC machine.

The second step in the computations is to identify the "generating circles". This is accomplished by dealing with the tool in two consecutive positions. The movement of the tool between these positions is assumed to be linear. This is a valid assumption for small interpolation steps (less
than 1 mm). Between the two consecutive steps, the tool could move on a concave surface patch, on a convex patch, or on a flat surface. For any given tool path, like the one shown in Fig. 4, the parameters known at the two consecutive positions, \( i \) and \( i + 1 \), include the center of the tool, \( \overrightarrow{W}_i \), and a second point on the tool axis, \( \overrightarrow{P}_i \). The second point dictates the orientation of the tool and if not explicitly given, it can be generated from the orientation of the tool axis. All parameters are defined by vectors with respect to the global frame \( xyz \) as shown in Fig. 4. The corresponding points for the \((i + 1)th\) positions are \( \overrightarrow{W}_{i+1} \) and \( \overrightarrow{P}_{i+1} \). The location of the center of the inserts in this plane can be obtained from the following equation:

\[
\overrightarrow{M}_i = \frac{\overrightarrow{A} \times \overrightarrow{L}_i}{|\overrightarrow{A} \times \overrightarrow{L}_i|} \cdot \overrightarrow{R}
\]

(1)

where

\( \overrightarrow{A} \) is a unit vector along the fourth axis (rotational axis provided by a rotary table that rotates around the \( X \) axis), and, \( \overrightarrow{L}_i \) is a vector between \( \overrightarrow{P}_i \) and \( \overrightarrow{W}_i \). The vector between the center of the \( i^{th} \) and \((i + 1)^{th}\) inserts, \( \overrightarrow{C}_i \), can be obtained from:

\[
\overrightarrow{C}_i = (\overrightarrow{W}_{i+1} + \overrightarrow{M}_{i+1}) - (\overrightarrow{W}_i + \overrightarrow{M}_i)
\]

(2)

The location of the \( i^{th} \) point of contact is given by the vector \( \overrightarrow{Q}_i \).

\[
\overrightarrow{Q}_i = \overrightarrow{W}_i + \overrightarrow{M}_i + \overrightarrow{r}_i
\]

(3)
where

\( \overrightarrow{r}_i \) is a vector between the center of the insert and a contact point, and it is perpendicular to the vector \( \overrightarrow{C}_i \).

\[ \overrightarrow{r}_i \cdot \overrightarrow{C}_i = 0 \]  \( \text{(4)} \)

Depending on the geometry of contact between the tool and workpiece, there are three cases for determining the radius \( R_{\text{gen}} \) of the "generating circle". The first case, \( R_{\text{gen}} > R \); the contact points lie in the range \( 0^\circ \leq \beta < 90^\circ \) on both insert images, or the cut is performed by the outside of the torus. The second case, \( R_{\text{gen}} < R \); the contact points lie in the range \( 90^\circ < \beta \leq 180^\circ \) on both insert images, or the cut is performed by the inside of the torus. The last case is a special case where \( R_{\text{gen}} = R \); the tool axis is parallel in both positions, and \( \beta = 90^\circ \). In this case, the cut is performed by the bottom of the torus.

As shown in Fig. 4, the radius of the "generating circle" can be calculated as follows:

\[ R_{\text{gen}(i)} = |\overrightarrow{M}_i| + \psi \sqrt{|\overrightarrow{r}_i|^2 - \delta_i^2} \]  \( \text{(5)} \)

where

\( \psi \) is a parameter which depends on the surface characteristics and is given by:
\[
\psi = \begin{cases} 
+1 & \text{if the surface is concave} \\
-1 & \text{if the surface is convex} \\
0 & \text{if the point on the surface is an inflection point}
\end{cases}
\]

\(\delta_i\) is the projection of the vector \(\overrightarrow{r}_i\) on the vector \(\overrightarrow{L}_i\) and it can be calculated from:

\[
\delta_i = \frac{\overrightarrow{r}_i \cdot \overrightarrow{L}_i}{|\overrightarrow{L}_i|}
\]  \hspace{1cm} (6)

The equation of the "generating circle" in parametric form is then given by:

\[
\begin{align*}
(x) &= A_i R_{gen} \cos(\omega) + B_i R_{gen} \sin(\omega) \\
y &= A_i R_{gen} \cos(\omega) + B_i R_{gen} \sin(\omega) \\
z &= A_i R_{gen} \cos(\omega) + B_i R_{gen} \sin(\omega)
\end{align*}
\]  \hspace{1cm} (7)

where

- \(\omega\) is the parameter which is actually the angle swept as we move around the circumference,
- \(A\) is vector along the center of the rotary table,
- \(B\) is a unit vector parallel to the direction of feed and lies in the cutting plane, and,
- \(R_{gen}\) is the radius of the "generating circle".

Figure 5a shows an example of the "generating circles" in four subsequent tool positions. These circles describe the bottom surface of the swept volume. To generate the sides of that volume, the portion of the cutting surface of the inserts above these circles must also be included as illustrated in Fig. 5. The cutting surface up to the center of the torus needs only to be considered.

Other than in very simple situations, dealing with the whole "generating circles" shown in Fig. 5a poses serious difficulties in determining the swept volume; the circles intersect with each other in such a way that it becomes difficult to discern what is on the outside or on the inside of the swept surface. To alleviate this problem, the "generating circle" is split into two halves, the front and the back to generate two swept volumes as shown in Fig. 5b,c. Both swept volumes have to be combined using Boolean operations to get the final swept volume. The increased number of operations involved in treating halves of the "generating circle" was deemed necessary to offset the difficulties in dealing with the whole circle, and thus to make the developed technique more robust.

The "generating circles" described above are determined at all intermediate points along the tool path. The start and end positions are treated differently. At the starting position, the "generating circles" radius is simply \(R\). At the end position, the radius of the "generating circle" is calculated based on the last two points of the given tool path (only for the part generated by the back of the tool). The toroidal cutting surface (only the bottom half) is split into 2 halves, the back half is added to the surface in Fig. 5c at the start point, and the front half is added to the surface in Fig. 5b at the end point. These surfaces are used to generate the swept volumes, by dividing the circumference of all semi-circles, including the profile of the inserts, into segments, and con-
necting the same point in two subsequent tool positions by a straight line. In this way the bottom surfaces (front and back) swept by the toroidal cutter from start to finish are fully described. Figure 6 is an exploded diagram of an example swept volume, it shows the bottom surface swept by the "generating circles", the front of the toroidal cutting surface at the end of the cut, the back

Fig. 5. An example of the "generating circles" in consecutive tool positions.

Fig. 6. Various components of swept volume of an example tool path.
toroidal surface at the beginning of the cut, and the sides swept by the inserts. This figure also
shows the volume swept by the cylindrical shank of the tool.

Creation of generic primitives requires access to low level Euler functions in a solid modelling
system. For this work, an in-house solid modeller (GWB) based on the work of Mantyla [17] was
used. This modeller provides access to low level Euler functions and can be compiled with a
custom "C" code.

4. Experimental verifications

The proposed technique was used to simulate 3 cuts: a circular, a spiral, and an inclined sinus-
oidal wave.

In the circular cut the toroidal cutter was rotated by 45° around a fixed point on the tool axis.
The radius of the arc was 88.65 mm, and the tool involved had the dimensions, R = 12.7 mm,
and r = 6.35 mm. In this very special case the radius of the generating circle, the effective radius,
remains constant. In effect, it is like using the same flat end mill of that radius for the entire cut.
Figure 7 shows the simulated swept volume and the resulting workpiece.

The spiral cut was performed exactly like the circular cut, except the tool moved uniformly
upward for a distance of 10 mm in the Z direction during the rotation of axis A. This then caused
a linear variation in the radius of the generating circle.

The previous two cuts were very easy to visualize and to predict the resulting workpiece surface.
The third cut, the sinusoidal cut, was more challenging. In this cut, the axes X, Y, Z and A moved
simultaneously to generate an inclined cosine wave, of wave length 100 mm, and an amplitude
of 5 mm. The travel in the X direction for this cut was 50 mm. The bottom center of the tool
was programmed to move along this cosine wave while ensuring that the bottom plane of the
tool remained tangent to the curve as shown in Fig. 8. Because of symmetry, only half of the
tool path is shown in Fig. 8. Unlike the first two examples of simulating concave surfaces, the
current example includes concave and convex surfaces as well as an inflection point. Because of
the way the tool was deliberately programmed, gouging occurred everywhere, and the resulting
swept volume was generated partially by the front of the tool and partially by the back of the
tool. The final outcome would by unknown a priori and the radius of the generating circle would
change from point to point in an unknown fashion. The swept volume for this complicated cut
is shown in Fig. 9.

The three tool paths simulated above were also used in machining of three test pieces by using
an OKK MCV 410 4 axis machine, with 3 translational axes X, Y, and Z, and a rotary axis A.
The workpiece was mounted in an overhang position in a fixture clamped to the rotary table.
This available arrangement was not particularly rigid, and for this reason machining wax was
chosen for the workpiece material. The toroidal cutter used had 3 carbide inserts, and its dimen-
sions were R = 12.7, and r = 6.35 mm. The cutting conditions used were: spindle speed N =
1000 rpm, and feed = 150 mm/min.

The comparison between the simulation and the cutting tests were performed in terms of the
Z coordinates of points on the workpiece obtained from the simulation with those measured on
the machined part. The comparison was conducted for the feed plane as well as for another section
in a plane parallel to the feed plane and offset from it by 6 mm. The measurement on the machined
part was conducted on a Mitotoyo BM 305 Coordinate Measuring Machine, CMM. Measurements were taken at 100 points along each section. It was not possible to match the locations of these points exactly with those generated in the simulation, and consequently some of the differences in the Z coordinates, albeit small, could be attributed to these deviations.

Figure 10 shows the comparison results for the circular path. The maximum difference is only 0.2 μm for the feed plane, and it is 0.8 μm for the offset plane. Although the agreement is excellent, yet it is expected because of the special nature of the cut.

Figure 11 shows the comparison results for the spiral cut. The maximum difference is 0.3 μm for the middle section, and it is about 1 μm for the offset section. Again the differences are very small and they do not follow any specific pattern. In this tool path, however, we should remember
Fig. 8. The sinusoidal tool path.

Fig. 9. The solid model of the volume swept by the tool for the sinusoidal cut.
Fig. 10. The difference between measured and simulated results for the circular cut at two sections.

Fig. 11. The difference between measured and simulated results for the spiral cut at two sections.

that the radius of the generating circle increased linearly from the beginning to the end of cut. Such a trend did not manifest itself in the comparison results, and accordingly the differences in Fig. 11 could be attributed mainly to experimental errors.

Figure 12 shows the comparison results for the sinusoidal tool path. Because of the complexity of this tool path, comparisons were made at three sections; the middle, a plane offset by 6 mm, and a plane offset by 10 mm. The maximum difference at all sections is about 8 μm. This is an order of magnitude greater than the previous two examples. Nevertheless, it is still too small, especially considering the fact that typical tolerances for sculptured surfaces are around ± 50 μm.
Fig. 12. The difference between measured and simulated results for the sinusoidal cut at three sections.

Again, there is no apparent pattern in the results of Fig. 12, which means that the observed differences are due to random errors rather than systematic deviations associated with the proposed technique.

5. Comparison with another simulator

The experimental results presented in Section 4 indeed support the proposed technique for simulating swept volumes using the generating circles. The differences between simulated and measured results could be attributed mainly to experimental errors, especially for the sinusoidal cut where all the 4 axes moved simultaneously, and the mounting overhang was large. To eliminate these potential sources of errors, numerical comparisons were also conducted between the proposed technique, and another simulation technique known as the Z-map method.

The Z-map simulation has been established and used to determine surface finish in ref. [14]. Rao et al. [15] also used it to determine gouging and to generate gouge free tool path. The details of the method are described in ref. [18], but a summary of the technique is given here. In the Z-map method, vertical rays (grass) of a specified height are constructed at closely spaced points along the region to be simulated. These rays are clipped by the tool as it moves from the starting point to the end point. At the end of the path, the height of the rays gives a good assessment of the existing surfaces. This method was implemented and used to simulate the resulting workpiece surface along the feed plane section, as well as for other offset sections. The grass density was 100 per mm, and the distance between the subsequent tool positions was 0.35 mm.

For both the circular and spiral tool paths, the agreement between the generating circles method
and the Z-map method was almost perfect; differences were less than 0.1 μm. Fig. 13 shows the differences between the two simulation techniques for the sinusoidal cut. They are within ±1 μm, which provides more evidence that the proposed technique is indeed a viable one to generate swept volumes in multi-axis machining.

6. Discussions

This paper has presented a novel approach for calculating the swept volume of toroidal cutters. This approach has been supported experimentally and through numerical comparisons with the Z-map method. The objective of developing the new approach is to speed up the tool path simulation. The comparison with the Z-map method revealed that, the computation time, associated with the sinusoidal cut was 9 seconds for the new technique versus 27 seconds for the Z-map. We should keep in mind that in the new approach, the whole imprint of the tool onto the workpiece surface was computed, whereas in the Z-map method the surface was simulated at 3 sections only. Computations were done in C on a Sun sprac 20 workstation. Another comparison was made for the sinusoidal cut, in which Boolean operations were used at each tool position to simulate the resulting surface. The simulation was conducted using the package ACIS on a Pentium 75 MHz PC. It took 14 hours of computing time to generate half of the cosine curve. Even if we consider a factor of 50 for fair comparison taking into account the computing platforms, still the balance tilts heavily in favor of the generating curves. It was not possible to compare the computations times involved in the new technique with all other applied simulation methods; that would have unnecessarily increased the scope of the current work.

So far, the new approach has been tried out on cuts in a single plane that includes the tool axis. Treating general tool paths where movements of the tool would involve twists and turns

![Fig. 13. A comparison between the “generating circles” technique and the Z-map technique](image-url)
(out of plane) will be subject for future research. Also an investigation is needed to study the effect of the step size (the distance in the feed direction between two subsequent tool positions) on the accuracy of the simulated swept volume. In the present study a step size of only 0.35 mm was used, and that resulted in very small errors between the simulated and measured surfaces. It would be tempting to speed up the simulation further by increasing the step size with the constraint being the required tolerance on the surface considered. Such investigation would involve the study of a wide range of surfaces to arrive at some recommended values to be used in a CAM package.

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