Problem 1
Bedford & Fowler, © 2005 by Pearson Education Inc.

Given:
• The 20-kg mass is suspended by cables attached to three vertical 2-m posts.
• Point A is \((0, 1.2, 0)\) m.

Goal:
Determine the tensions in cables AB, AC and AD

Solution:

Step 1: FBD

The weight of the mass is:
\[
W = (20\text{kg}) \left( 9.81 \text{ m/s}^2 \right) = 196.2\text{ N}
\]

Step 2: Find the coordinates of the points
\[A = (0, 1.2, 0), \quad B = (-0.3, 2, 1), \quad C = (0, 2, -1), \quad D = (2, 2, 0)\]

Step 3: Write the position vectors
\[
\vec{r}_{AB} = -0.3\hat{i} + 0.8\hat{j} + \hat{k}
\]
\[
\vec{r}_{AC} = 0\hat{i} + 0.8\hat{j} - \hat{k}
\]
\[
\vec{r}_{AD} = 2\hat{i} + 0.8\hat{j} + 0\hat{k}
\]
Step 4: Write the unit vectors in the direction of the forces

\[
\hat{u}_{AB} = \frac{-0.3\hat{i} + 0.8\hat{j} + \hat{k}}{\sqrt{(-0.3)^2 + (0.8)^2 + (1)^2}} = -0.228\hat{i} + 0.608\hat{j} + 0.760\hat{k}
\]

\[
\hat{u}_{AC} = \frac{0\hat{i} + 0.8\hat{j} - \hat{k}}{\sqrt{(0)^2 + (0.8)^2 + (-1)^2}} = 0\hat{i} + 0.625\hat{j} - 0.781\hat{k}
\]

\[
\hat{u}_{AD} = \frac{2\hat{i} + 0.8\hat{j} + 0\hat{k}}{\sqrt{(2)^2 + (0.8)^2 + (0)^2}} = 0.928\hat{i} + 0.371\hat{j} + 0\hat{k}
\]

Step 5: Write the Cartesian form of the forces

\[
\vec{F}_{AB} = -0.228\vec{F}_{AB}\hat{i} + 0.608\vec{F}_{AB}\hat{j} + 0.760\vec{F}_{AB}\hat{k}
\]

\[
\vec{F}_{AC} = 0.625\vec{F}_{AC}\hat{j} - 0.781\vec{F}_{AC}\hat{k}
\]

\[
\vec{F}_{AD} = 0.928\vec{F}_{AD}\hat{i} + 0.371\vec{F}_{AD}\hat{j}
\]

\[W = -196.2\hat{j}\]

Step 6: Apply Equilibrium Equations

\[
\sum F_x = 0 : \quad -0.228|\vec{F}_{AB}| + 0 + 0.928|\vec{F}_{AD}| = 0 \quad (1)
\]

\[
\sum F_y = 0 : \quad 0.608|\vec{F}_{AB}| + 0.625|\vec{F}_{AC}| + 0.371|\vec{F}_{AD}| - 196.2 = 0 \quad (2)
\]

\[
\sum F_z = 0 : \quad 0.760|\vec{F}_{AB}| - 0.781|\vec{F}_{AC}| + 0 = 0 \quad (3)
\]

Step 7: Solve the equations

From (3):

\[
|\vec{F}_{AC}| = \frac{0.760}{0.781}|\vec{F}_{AB}|
\]

From (1):

\[
|\vec{F}_{AD}| = \frac{0.228}{0.928}|\vec{F}_{AB}|
\]

Into (2):

\[
0.608|\vec{F}_{AB}| + (0.625)\left(\frac{0.760}{0.781}\right)|\vec{F}_{AB}| + (0.371)\left(\frac{0.228}{0.928}\right)|\vec{F}_{AB}| - 196.2 = 0
\]

Hence, \[|\vec{F}_{AB}| = 150\text{kN}, \quad |\vec{F}_{AC}| = 146\text{kN}, \quad |\vec{F}_{AD}| = 36.9\text{kN}\]
Problem 2

Bedford & Fowler, © 2005 by Pearson Education Inc.

The force $\mathbf{F}$ exerted on the grip of the exercise machine point in the direction of the unit vector $\hat{u} = \frac{2}{3} \mathbf{i} - \frac{2}{3} \mathbf{j} + \frac{1}{3} \mathbf{k}$ and its magnitude is 120N.

Goal:

(a) Determine the magnitude of the moment of $\mathbf{F}$ about the origin $O$.

The support will safely support a moment of 560N-m magnitude. Based on this criterion

(b) What is the largest safe magnitude of $\mathbf{F}$?

(c) If the force $\mathbf{F}$ may be exerted in any direction, what is its largest magnitude?

SOLUTION:

(a) The vector from $O$ to the point of application of the force is

$$\mathbf{r} = 0.25 \mathbf{i} + 0.2 \mathbf{j} - 0.15 \mathbf{k} \text{ m}$$

The force is $\mathbf{F} = |\mathbf{F}| \hat{e}$. Therefore,

$$\mathbf{F} = 80 \mathbf{i} - 80 \mathbf{j} + 40 \mathbf{k} \text{ N}$$

The moment of $\mathbf{F}$ about $O$ is:

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.25 & 0.2 & -0.15 \\ 80 & -80 & 40 \end{vmatrix} \text{ N-m}$$

or $\mathbf{M}_O = -4 \mathbf{i} - 22 \mathbf{j} - 36 \mathbf{k} \text{ N-m}$, and

$$|\mathbf{M}_O| = \sqrt{4^2 + 22^2 + 36^2} = 42.4 \text{ N-m}$$
(b) Since we do not know the force exerted, the moment can be written as,

\[ \overrightarrow{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.25 & 0.2 & -0.15 \\ \frac{2}{3} \overrightarrow{F} & -\frac{2}{3} \overrightarrow{F} & \frac{1}{3} \overrightarrow{F} \end{vmatrix} \]

Hence,

\[ \overrightarrow{M}_O = (0.0333 \hat{i} - 0.1833 \hat{j} - 0.3 \hat{k}) |\overrightarrow{F}| \]

And the magnitude of \( \overrightarrow{M}_O \) is

\[ |\overrightarrow{M}_O| = \sqrt{(0.0333)^2 + (-0.1833)^2 + (-0.3)^2} |\overrightarrow{F}| = 0.353 |\overrightarrow{F}| \]

If \( |\overrightarrow{M}_O| \leq 560 \text{ N-m} \)

\[ 560 = 0.353 |\overrightarrow{F}_{\text{max}}| \Rightarrow |\overrightarrow{F}_{\text{max}}| = 1.59 \text{ kN} \]

(c) If \( \vec{F} \) can be in any direction, then the worst case is when \( \vec{r} \perp \vec{F} \)

If the magnitude of \( \vec{r} \) is given by:

\[ |\vec{r}| = \sqrt{(0.25)^2 + (0.2)^2 + (-0.15)^2} = 0.3536 \text{ m} \]

The largest magnitude of the force \( \vec{F} \) is given by:

\[ 560 = |\vec{r}| |\overrightarrow{F}_{\text{worse}}| \Rightarrow |\overrightarrow{F}_{\text{worse}}| = 1.58 \text{ kN} \]
Problem 3


Given:
- A fixed crane has a mass of 1000kg and is used to lift a 2400kg crate.
- It is held in place by a pin at A and a rocker at B.
- The center of gravity of the crane is located at G.

Goal:
Determine the components of the reactions at A and B

Solution:

Free-Body Diagram:

Obtain the weights of the crane and the crate:

\[ W_{\text{crane}} = (1000\text{kg}) \left( \frac{9.81\text{m}}{\text{s}^2} \right) \]

\[ W_{\text{crane}} = 9.81\text{kN} \]

\[ W_{\text{crate}} = (2400\text{kg}) \left( \frac{9.81\text{m}}{\text{s}^2} \right) \]

\[ W_{\text{crate}} = 23.5\text{kN} \]

Determination of B:

\[ + \sum M_A = 0 : \]

\[ +B(1.5\text{m}) - (9.81\text{kN})(2\text{m}) - (23.51\text{kN})(6\text{m}) = 0 \]

\[ B = +107.1\text{kN} \text{ or } B = 107\text{kN} \rightarrow \]
Determination of $A_x$:

$\sum F_x = 0$

$A_x + B = 0$

$A_x = -107.1\, \text{kN}$ or $A_x = 107\, \text{kN}$ ←

Determination of $A_y$:

$\sum F_y = 0$

$A_y - 9.81\, \text{kN} - 23.5\, \text{kN} = 0$

$A_y = 33.3\, \text{kN}$ or $A_y = 33.3\, \text{kN}$ ↑

Adding vectorially the components $A_x$ and $A_y$:

$|\vec{A}| = \sqrt{(107.1)^2 + (33.3)^2} \approx 112\, \text{kN}$

$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{33.3}{107.1}\right) \approx 17.3^\circ$ measured from $A_x$ to $A_y$
Problem 4

The meshed gears are subjected to the couple moments shown.

Goal:
Determine the magnitude of the resultant couple moment and specify its coordinate direction angles.

Solution:

\[
\begin{align*}
\vec{M}_1 &= \{50 \hat{k}\} \text{N.m} \\
\vec{M}_2 &= 20 \left( -\cos 20^\circ \sin 30^\circ \hat{i} - \cos 20^\circ \cos 30^\circ \hat{j} + \sin 20^\circ \hat{k} \right) \text{N.m} \\
\vec{M}_2 &= \{-9.397 \hat{i} - 16.276 \hat{j} + 6.840 \hat{k}\} \text{N.m}
\end{align*}
\]

The resultant couple moment is then:

\[
\vec{M}_R = \sum \vec{M} : \vec{M}_R = \vec{M}_1 + \vec{M}_2
\]

\[
\vec{M}_R = \{50 \hat{k}\} \text{N.m} + \{-9.397 \hat{i} - 16.276 \hat{j} + 6.840 \hat{k}\} \text{N.m}
\]

\[
\vec{M}_R = \{-9.397 \hat{i} - 16.276 \hat{j} + 56.840 \hat{k}\} \text{N.m}
\]

The magnitude of the resultant couple moment is:

\[
M_R = \sqrt{(-9.397)^2 + (-16.276)^2 + (56.840)^2}
\]

\[
M_R = 59.867 \text{N.m} = 59.9 \text{N.m}
\]

The coordinate direction angles are:

\[
\alpha = \cos^{-1} \left( \frac{-9.397}{59.867} \right) = 99.0^\circ, \quad \beta = \cos^{-1} \left( \frac{-16.276}{59.867} \right) = 106^\circ, \quad \gamma = \cos^{-1} \left( \frac{56.840}{59.867} \right) = 18.3^\circ
\]