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ABSTRACT (350 words)

A design approach for an aircraft takeoff performance monitoring system (TOPMS) is described. In this approach, it is proposed that the Global Positioning System (GPS) be used to determine aircraft acceleration, ground speed, and position relative to the end of the runway. A practical evaluation of the feasibility of this proposal showed clear superiority of a GPS-derived acceleration over a more traditional method employing accelerometers. This study found that, when compared to measurements from carefully mounted accelerometers, the GPS-derived measurement agreed to within 0.10 metres per second squared 90% of the time. Advantages of the GPS-derived measurement include a modest noise level, insusceptibility to gravity and temperature-influenced variations, and far simplified mounting criteria.

A theoretical dynamic model of an aircraft in contact with the ground was devised. A GPS receiver and data acquisition system was designed and certified, then installed in an aircraft operated by an airline servicing far-northern Canadian airports. The data collected in this manner were used to validate the theoretical model. It was concluded that a projection of displacement can be determined to within an uncertainty of fifteen metres in sufficient time to alert the pilot of an unsafe situation.
ACKNOWLEDGEMENTS

I will begin by acknowledging the corporate sponsorship of Transwest Air. Their willingness to provide space aboard a passenger aircraft for a prototype device enabled the research conducted during this project to be much more relevant than would have been the case with an economical alternative. Partners such as Transwest are rare and invaluable.

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Dan Aspel and Glenn Wright are friends who have served as my personal navigation system over the past few years. As with any occupation, or preoccupation, that consumes so much of one’s time, friends are often consulted for reassurance regarding the usefulness of the undertaking. I sincerely appreciated the patience of those who acted as my sounding board, as well as the advice that was constantly available.

I would like to thank my family for their support and understanding. While I credit my parents with teaching me to question everything that is generally considered to be factual, an approach which has served me very well in technical matters, they know too well that this particular trait often has a downside. I would like to express my sincere appreciation to my entire family for their patience.

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DEDICATION

I have been told that the one common desire of people the world over is to leave the world a slightly better place for the next generation to inherit.

For Nathan.
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NOMENCLATURE

Lowercase Variables and Vectors

\( a \) \quad \text{the component of acceleration in the tangential or subscripted direction}
\( a^* \) \quad \text{the measurement from an accelerometer}
\( c \) \quad \text{the speed of light}
\( \vec{e} \) \quad \text{a vector of state estimate errors}
\( \vec{e}^- \) \quad \text{a vector of projected state estimate errors}
\( g \) \quad \text{the gravitational constant}
\( j \) \quad \text{the tangential jerk, where jerk is the first time derivative of acceleration}
\( m \) \quad \text{the mass of an object}
\( q \) \quad \text{a mean-zero random variable describing process noise}
\( r \) \quad \text{a mean-zero random variable describing sensor noise}
\( s \) \quad \text{one-dimensional displacement}
\( t \) \quad \text{the time elapsed since a reference event}
\( v \) \quad \text{the tangential speed}
\( v_a \) \quad \text{the component of wind velocity in the direction of vehicle motion}
\( v_{1,a} \) \quad \text{the speed with reference to a moving mass of air}
\( \vec{w}_1 \) \quad \text{the process noise in a state space representation}
\( \vec{w}_2 \) \quad \text{the sensor noise in a state space representation}
\( x \) \quad \text{the displacement in the x-direction of a coordinate system}
\( \vec{x} \) \quad \text{a vector of state variables in a state space representation}
\( \hat{x} \) \quad \text{a vector of state estimates}
\( \hat{x}^- \) \quad \text{a vector of state estimate projections}
\( y \) \quad \text{the displacement in the y-direction of a coordinate system}
\( \vec{y} \) \quad \text{a vector of measurement variables in a state space representation}
\( \hat{y} \) \quad \text{a vector of measurement estimates}
\( z \) \quad \text{the displacement in the z-direction of a coordinate system}
Matrices

A  the state transition matrix in a state space representation
C  a matrix relating measurements to states in a state space representation
I  the identity matrix
K  the observer gain matrix
Q  the state covaraince matrix in a state space representation
R  the measurement covariance matrix in a state space representation

Uppercase Variables

A  frontal area
D  the force due to aerodynamic drag
\( D_n \)  constants in a function modelling aerodynamic drag
\( E \{ \} \)  the expected value of an expression containing random variables
F  the force due to friction
\( F_n \)  constants in a function modelling friction
L_1  the primary GPS carrier frequency
L_2  the secondary GPS carrier frequency
P_n  constant parameters modelling vehicle dynamics
T  the force due to engine thrust
\( T_n \)  constants in a function modelling engine thrust
V_1  critical engine failure recognition speed
W  the force due to weight
W_1  the sine of the angle describing the inclination of a surface

Greek Variables

\( \rho_{\text{air}} \)  the density of air

Superscripts

\( T \)  superscript indicating the transpose of the superscripted matrix
Subscripts

\( k \) subscript identifying the “current” time step in a discrete filter

\( k + 1 \) subscript identifying the “next” time step in a discrete filter

\( x \) subscript identifying the component of a quantity in the x-direction

Acronyms and Abbreviations

- **ADSR** Accelerate-Stop Distance Required
- **ATR**
- **C/A** Coarse/Acquisition
- **DGPS** Differential GPS
- **FLL** Frequency Locked Loop
- **GPDR** Global Positioning Data Recorder
- **GPS** Global Positioning System
- **IMU** Inertial Measurement Unit
- **INS** Inertial Navigation System
- **LDR** Landing Distance Required
- **Mcps** Megachips per Second
- **MCU**
- **NASA** National Aeronautics and Space Administration
- **PLL** Phase Locked Loop
- **PRN** Pseudorandom Noise
- **SA** Selective Availability
- **SV** Space Vehicle
- **TCAS** Traffic/Collision Avoidance System
- **TNC**
- **TOPMS** Takeoff Performance Monitoring System
- **VDC** Volts, Direct Current
Chapter 1 - Introduction

1.1 Aircraft Landing and Takeoff Performance Monitoring

Aircraft landing and takeoff performance monitoring is an area of research aimed at improving the information available to the pilot for decision making during takeoff or landing. A system capable of instantaneously determining the stopping distance of an aircraft could form an integral component of a monitoring system. Particularly difficult to quantify is the frictional coefficient between the runway and the aircraft tires, should such a measurement be necessary. In secluded far-northern regions, where a monitoring system would be particularly useful given adverse weather, few airports are equipped to attempt frictional measurements. In such instances, a monitoring system would need to be totally self-contained and able to determine aircraft ground speed, acceleration, and position relative to the end of the runway with reference to a theoretical dynamic model relating these parameters. Prediction of the aircraft's location at rest is then possible.

Landing and takeoff performance monitoring systems are aimed at averting runway overrun when an aircraft is in contact with the ground. Typical causes of runway overrun include engine failure on takeoff and reduced braking resulting from runway contamination. In northern regions, this has been identified as a common problem.

The “critical engine failure recognition speed” \( (V_1) \) is defined as the speed above which
takeoff could continue safely if the most critical engine failed,\textsuperscript{1} assuming the runway length is sufficient. $V_1$ is often calculated prior to startup based on aircraft parameters and estimation of runway and weather conditions. Choosing a throttle setting to reach $V_1$ is a more complicated matter. With a low throttle setting, takeoff rejection initiated at a speed slightly below $V_1$ may result in runway overrun, while a high power setting increases the likelihood of an engine failure on takeoff. As well, engine service life depends largely on its peak power setting.

In a theoretical rejected takeoff, the aircraft accelerates gradually until the rejection is initiated. Drag increases with airspeed, giving rise to lower accelerations at higher airspeed. Once the rejection is initiated, the aircraft decelerates gradually to rest. In practicality, there would be a measurable reaction time delaying the application of braking and reverse thrust.

Operators often use the so called “balanced field concept” to calculate the lowest possible power setting for use during takeoff. Then, at speeds below $V_1$, there is mathematically enough runway remaining to abort the takeoff. Once $V_1$ is reached, the aircraft could safely takeoff even in the event of the failure of one engine. With this in mind, $V_1$ becomes a “decision” speed. Figure 1.1 shows this scenario with a takeoff rejection initiated at a decision speed of eighty metres per second on a 2400-metre runway. In reality, pilots refer to performance charts to determine power settings and decision speeds. The degree of uncertainty present in this method is significant. Consequently, pilots have little confidence in the utility of such charts at times when decisions must be made
quickly.² Figure 1.2 shows the results of a takeoff rejected an instant after reaching $V_1$.

**Figure 1.1** Theoretical Rejected Takeoff

Performance monitoring systems³ that could provide to the pilot information pertaining to the level of safety with which a takeoff is proceeding are currently in existence, but have yet to be adopted by manufacturers. In such systems, the pilot is required to provide overall runway length information as well as runway frictional coefficient data based on measurements periodically taken using ground-based vehicle-mounted measurement
systems. The level of error in such measurements can be significant.

Figure 1.2 Rejected Takeoff of Canadian Airlines International Flight 17

A similar system for use during approach and landing is currently unavailable because of the inability for the pilot to provide remaining runway length. The Global Positioning System (GPS) could be used to determine remaining runway length. The same observer system could then be used for both takeoff and landing.

1.2 The Far-Northern Environment

The runway overrun problem is further aggravated in inclement weather where runway surfaces are contaminated by water or ice. Far-northern regions experience this sort of
climate over six months of the year. Further, as such regions are relatively less populated, facilities may receive infrequent maintenance. Accounting for these factors in the landing or takeoff decision-making process represents a significant challenge for pilots.

Many airports in far-northern regions are gravel surfaced. The behaviour of a gravel runway may be unpredictable, especially when temperatures are near the freezing point. Measurements of runway friction attempted in such conditions would be relatively unreliable.

The availability of radio navigation systems in far-northern regions is also an issue. While such facilities exist, they are sparsely distributed and tend to service the airports of major population centres. Air carriers that service airports in support of mining and forestry are less likely to have reliable access to radio navigation facilities. Navigational information provided by GPS receivers is now available in these areas and has enhanced or, in some cases, replaced existing facilities.
Chapter 2 - Literature Review

2.1 Background - Aircraft Landing and Takeoff Safety

One of the most common aviation accident events continues to be runway overrun during takeoff or landing. In the case of takeoff runway overrun, the problem is often associated with engine power loss. This problem is further aggravated in inclement weather where runway surfaces are contaminated by water or ice. Pilots of multi-engine aircraft must evaluate a complex set of variables in situations involving varying winds, limited control of ground traction, and necessary application of reverse thrust.

2.1.1 Early Research in Takeoff Performance Monitoring

In 1984, the Society of Automotive Engineers began drafting a specification to govern the design standards for takeoff performance monitors. In 1985, Raghavachari Srivatsan authored a doctoral thesis at the University of Kansas regarding the design of a Takeoff Performance Monitoring System (TOPMS). He continued work on this project and, by 1987, such a system was developed at NASA’s Langley Research Centre for potential implementation in Boeing’s B777 shown in figure 2.1. Simulator evaluations were completed in 1992, and flight testing was performed in 1994. The proposal to include the instrument in the B777 was inevitably rejected due to practical shortcomings. Specifically, there was concern over the non-predictability and variability of wind and runway conditions and the manner in which the device would compensate for this lack of
information. Manufacturers feared that the device may do more harm than good, possibly distracting the pilot unnecessarily.

Figure 2.1 Boeing B-777

Concern over unpredictable conditions is understandable. On dry, paved runways, the primary means of deceleration for a large jet aircraft is the application of wheel braking. Reverse thrust is available, but accounts for only about 20% of the force required for deceleration. Estimating the maximum braking force available is by no means trivial. While the condition of the runway is a factor, several factors unique to each aircraft are important. This adds to the uncertainty in any projected stopping distance.

In the design of his TOPMS,\textsuperscript{10} Srivatsan accounted for a litany of variables that influence
the distance required for an aircraft to accelerate or decelerate, as appropriate. These included the ambient air pressure and temperature, the weight of the aircraft and its centre of gravity, the flap setting, the pitch attitude, the throttle setting and engine pressure ratio, the wind speed and direction, the rolling friction coefficient, the acceleration of the aircraft, and the calibrated airspeed. Additionally, those parameters that change throughout the takeoff roll were referenced to theoretical models. The net result of this treatment was a large uncertainty in the predicted takeoff roll, as much as 5% of the overall displacement of the aircraft. Moreover, the required instrumentation limited the applicability of the design to large passenger jet aircraft.

In Canada, engine failures occur during one in every 76,000 jet takeoffs\textsuperscript{11}. While this frequency is significantly high to warrant TOPMS research, it demands that the instrument be carefully designed to avoid nuisance alerts that would recommend takeoff rejection in borderline cases where the takeoff need not be rejected.

Several other performance monitoring systems\textsuperscript{12,13} have been devised. In most designs, the pilot would have been required to manually supply runway length information as well as runway frictional coefficient provided by ground based observers. No large-scale implementation of such a device has been published. There would be much more likelihood of implementation were the system completely self-contained. More importantly, there has been no published work on a monitor specifically intended for use in the unique far-northern environment.
2.1.2 Regulatory Issues

In Canada, there are currently no regulations regarding the procedures to be followed when conducting a takeoff or landing on a gravel runway. Some aircraft performance charts include information pertaining to gravel runways, but manufacturers are not required to provide such information. As a result, operators are left with performance charts pertaining to dry, paved runways. Moreover, in compiling such performance charts it is typically assumed that reverse thrust is unavailable. While it would be prudent to account for the reduced utility of wheel brakes on gravel runways by extending the required runway length or reducing the aircraft payload, such measures carry financial implications for operators.

At a preliminary meeting of officials at Transport Canada, some guidelines were established in pursuit of regulations specifically intended to govern the use of gravel runways. While it is common knowledge in the industry that the primary means of deceleration on gravel runways is through the application of reverse thrust, the preliminary guidelines state that “no credit for propeller reverse may be used in calculation of Accelerate - Stop Distance Required (ASDR) or Landing Distance Required (LDR).” To paraphrase, only wheel braking may be considered in a determination of the required runway length for takeoff or landing.

2.2 The Global Positioning System

The Global Positioning System is a satellite navigation system that provides a means of calculating time, position, and velocity data using coded signals which can be processed
using a receiver. A minimum of four satellite signals is used to compute three-dimensional positions. A GPS receiver derives position information by measuring the time required for a signal to be transmitted from a satellite at a known position. The range measurements for each satellite are the product of the speed of light, \( c \), and the time required for the signal to travel,

\[
c(t-t_{1}) = \sqrt{(x-x_{1})^{2} + (y-y_{1})^{2} + (z-z_{1})^{2}}, \quad (2.1)
\]

\[
c(t-t_{2}) = \sqrt{(x-x_{2})^{2} + (y-y_{2})^{2} + (z-z_{2})^{2}}, \quad (2.2)
\]

\[
c(t-t_{3}) = \sqrt{(x-x_{3})^{2} + (y-y_{3})^{2} + (z-z_{3})^{2}}, \quad (2.3)
\]

\[
c(t-t_{4}) = \sqrt{(x-x_{4})^{2} + (y-y_{4})^{2} + (z-z_{4})^{2}}, \quad (2.4)
\]

where: \( x_n, y_n, z_n, t_n \) are known satellite positions and times of signal transmission,

and;

\( x, y, z, t \) are the receiver location and times of receipt of each signal.

There are several sources of inaccuracy in this process including receiver noise, tropospheric delay, multipath error, satellite clock errors, orbit errors, and ionospheric delay. Until May 1, 2000 the United States Department of Defense injected intentional degradation or Selective Availability (SA) into the transmitted signal for security reasons. At the beginning of this project, SA was by far the largest contribution to position error,
on the order of one hundred metres. However, this error could be described as a slow wandering bias error. The resulting velocity error from time differentiation was less than one metre per second. Further, the velocity error changed slowly resulting in a virtually negligible acceleration error. The remaining error sources contribute a relatively steady bias error on the order of a few metres. With the exception of receiver noise and multipath error, these errors were highly repeatable when considering time intervals of less than one second. Multipath error occurs when reflected GPS signals are misinterpreted by the GPS receiver as having come directly from the satellite. Airport runways are typically low-multipath environments a clear view of the sky is generally available and because buildings are not within close proximity. With no multipath error and SA off, position accuracy of less than ten metres is possible.

It is possible to compensate for errors other than multipath and receiver noise using Differential GPS (DGPS). The concept of DGPS involves the use of a stationary GPS receiver at a known location that is capable of transmitting corrections to a mobile receiver. Alternatively, such corrections can be stored and later used to improve data collected by a mobile receiver. Position accuracy of less than one metre, not counting the contribution of multipath error, can be achieved with DGPS if the distance between the two receivers is less than a few hundred kilometres.

The foundation for the determination of satellite range is the speed of light and the time required for transmission. The speed of light varies slightly as it passes through regions of the atmosphere, most importantly the ionosphere. The amount of the variability in speed
of light also depends on the frequency of the transmitted signal. As a result of this physical property, a GPS receiver can account for the ionospheric effect if it can receive GPS signals on two different frequencies. This capability was designed into the system from the beginning.

### 2.2.1 GPS Signal Structure

Originally intended for military use, the structure of the GPS transmitted signal was designed to provide a rapid means of calculating position and velocity. Two radio carrier frequencies were selected to carry signals from GPS satellites. The primary frequency, termed $L_1$, is centred at 1575.42 MHz. The secondary frequency, $L_2$, is centred at 1227.60 MHz. Each satellite transmits a unique signal at both frequencies, with a characteristic repeating digital code modulated on the carrier frequency. An example of the first 100 chips of one of the codes transmitted by space vehicle (SV) 24 is shown in figure 2.2. Each element in a code sequence is called a “chip” as opposed to a data bit because it is not actually data being transmitted in the code. The number of chips per second is called the chipping rate. The composition of the repeating digital code generated by each satellite can be varied based on operational requirements. Normal operation is described here.
Each satellite transmits satellite navigation data pertaining to all satellites at a rate of 50 bits per second. Part of these data represent the parameters in the equations of motion for each satellite. Typically, both the $L_1$ and $L_2$ signals contain this satellite navigation message. This information is used by the receiver to determine the positions of the satellites and to provide a rough approximation of the time and date, which can in turn be used to determine which satellites should be in view from an approximate geographic location. Because the satellite navigation message is transmitted at a relatively slow rate, it takes 12.5 minutes to receive the entire message. Each frame, corresponding to an individual satellite, takes thirty seconds to transmit. Consequently, it can take several minutes for a position fix to be calculated the first time a GPS receiver is activated.

The $L_1$ signal is also modulated with a repeating code, characteristic of the satellite from which it was sent, the purpose of which is to provide a coarse position determination. This code, called the coarse/acquisition (C/A) code, is a binary stream of 1023 chips.
transmitted at a rate of 1.023 Mcps so that the signal repeats once every millisecond. It is
the code that allows the GPS receiver to determine the time at which the signal was sent
from the satellite. A GPS receiver generates a reference signal to which the incoming
signal is compared. The receiver then adjusts its estimate of time of receipt until the
incoming signal and the reference signal are aligned. When this process is complete, the
receiver has determined the time, accurate to less than a microsecond, but with an
ambiguity of an integer multiple of a millisecond. One millisecond represents the time
required for light to travel 300 kilometres, so having observations from multiple satellites
resolves this ambiguity.

Both the L₁ and L₂ signal are also modulated with another repeating code, characteristic of
the satellite from which it was sent, the purpose of which is to provide a precise
determination of position. This binary data stream, called the P-code, is transmitted at
10.23 Mcps and repeats once every week. The receiver uses this code in a similar manner
to that described for the C/A-code. Receiving the signal on two different frequencies
allows the receiver to compensate for the change of the speed of light through the
atmosphere. The length of a P-code chip is also one tenth the length of a C/A-code chip,
which provides greater resolution.

The relative length of the individual constituent parts of the GPS signal provide an
indication of the available accuracy. The wavelength of the L₁ signal is 19.04 centimetres.
One P-code chip is 154 wavelengths or 29.33 metres. One C/A-code chip is 1540
wavelengths or 293.3 metres. Available accuracy is typically ten to fifty times better than
the chip size, so C/A-code tracking leads to P-code acquisition and P-code tracking leads to resolution of the carrier cycle.

### 2.2.2 GPS Signal Tracking

A GPS receiver determines velocity by measuring the Doppler shift of the incoming signal from the satellite. Because the range to each satellite is continuously changing, the frequency of the received signal is slightly different for each satellite. Important to note, for the purpose of later discussion, is the manner in which the incoming frequency is measured. A Frequency Locked Loop (FLL) is a signal tracking loop where the incoming signal is compared to a reference signal to arrive at an estimate of the incoming frequency. A Phase Locked Loop (PLL) is a signal tracking loop that incorporates the FLL capability but is intended to arrive at an estimate of the phase of the incoming signal. If the receiver has locked on to the phase of the incoming signal, then it has also locked on to the frequency. A block diagram for a PLL is shown in Figure 2.3. Phase error is fed back to a numerically controlled oscillator (NCO), which is essentially a firmware oscillator that estimates the carrier frequency and phase. The carrier frequency is the transmitted carrier frequency plus the Doppler shift resulting from the relative motion between the satellite and the receiver. The observed Doppler frequency can therefore be used to estimate receiver velocity and aid in propagating position measurements from one time step to the next.

Unless the signal tracking loop is aided by some external inertial sensor, the PLL (or FLL) treats any acceleration or higher order dynamics as an unmodelled disturbance. As a
result, using GPS-derived measurements to determine higher order parameters may suffer a performance penalty.

**Figure 2.3 Phase Locked Loop Block Diagram**

Once the PLL has removed the carrier from the signal, the resulting signal is the code from the satellite plus any remaining phase error. To determine range to each satellite, the GPS receiver generates the same code that the satellite has superimposed on the carrier signal. This reference code is then compared with the incoming signal in the Delay Locked Loop (DLL) shown in figure 2.4. The purpose of the DLL is to determine the amount of time by which the code is delayed from the receiver’s reference time. This measurement is used to determine the actual time with reference to the GPS datum, which is in turn used to determine the location of the satellite at the time of transmission. The location of the satellite and the time required for signal transmission is used to determine the range to the satellite and, eventually, the receiver position using equations (2.1) through (2.4).
2.3 State Observers

It is not always possible to directly measure parameters of importance to a particular system of states. For example, there is no means to directly measure the acceleration of a vehicle. It is possible to measure the net force applied to a vehicle, the orientation of that force with respect to the gravity vector, and the weight of the vehicle, and then infer the magnitude and direction of the acceleration of the vehicle. Alternatively, the speed or position of a vehicle can be measured over time and the acceleration can be determined through differentiation. However, direct differentiation amplifies any noise present in the measured quantity. “There are methods available to estimate unmeasurable state variables without a differentiation process. Estimation of unmeasurable state variables is commonly called observation.”16 A system of states is observable if the available measurements allow all states to be determined in a finite amount of time.

2.4 The Discrete Kalman Filter

The purpose of a Kalman Filter17 is to optimally estimate states in a theoretical model
based on sensor measurements. In a state-space representation such as the following, the state vector, $\mathbf{x}$, represents the actual condition of the system being observed, which includes process noise. The state transition matrix, $A$, need not be constant. Rather, it is the matrix which multiplied by the states at a time, $k$, would result in a noiseless state vector at a time, $k + 1$. Adding process noise, the states at $k + 1$,

$$
\mathbf{x}_{k+1} = A\mathbf{x}_k + \mathbf{w}_{1,k+1},
$$

are found, where the zero-mean process noise, $\mathbf{w}_1$, is described by the covariance matrix,

$$
\mathbf{Q} = E\{\mathbf{w}_1\mathbf{w}_1^T\}.
$$

The measurements,

$$
\mathbf{y} = C\mathbf{x} + \mathbf{w}_2,
$$

relate to the states through the matrix, $C$, in combination with zero-mean sensor noise, $\mathbf{w}_2$, described by the covariance matrix,

$$
\mathbf{R} = E\{\mathbf{w}_2\mathbf{w}_2^T\}.
$$

Once sensor measurements are taken, the state estimates computed in the prior time step, $\hat{x}^\sim$, can be improved based on how the measurements deviate from those the state transition matrix would project,

$$
\hat{x} = \hat{x}^\sim + K \left( \mathbf{y} - \hat{y}^\sim \right),
$$

18
or,

\[ \hat{x} = \hat{x}^+ + K \left( \bar{y} - C \hat{x}^+ \right), \tag{2.10} \]

where the observer gain matrix, \( K \), remains to be chosen. The optimal gain matrix is defined as that which would provide state estimates that deviate from the actual states by the least square error. To find this optimal gain, it is necessary to mathematically determine the variance of the state error. It is assumed that the process noise and the sensor noise are zero-mean random processes, so the variance of the state error should also be a zero-mean process.

The state error,

\[ \bar{e} = \bar{x} - \hat{x}, \tag{2.11} \]

expanded using (2.10),

\[ \bar{e} = \bar{x} - \hat{x}^- - K \left( \bar{y} - C \hat{x}^- \right), \tag{2.12} \]

and (2.7),

\[ \bar{e} = \bar{x} - \hat{x}^- - K \left( C \bar{x} + \bar{w}_2 - C \hat{x}^- \right) \tag{2.13} \]

then rearranging,

\[ \bar{e} = (\bar{x} - \hat{x}^-) - KC \left( \bar{x} - \hat{x}^- \right) - K\bar{w}_2 \tag{2.14} \]

and collecting terms,
\[ \bar{e} = (I - KC)(\bar{x} - \hat{x}) - K\bar{w}_2 \]  

(2.15)

or,

\[ \bar{e} = (I - KC)\bar{e} - K\bar{w}_2 \]  

(2.16)

provides a means to determine the error covariance,

\[ P = E\{\bar{e}\bar{e}^T\}. \]  

(2.17)

If there is no correlation between the process noise and the sensor noise, the expected values of the terms in (2.16) are separable. Thus,

\[ P = E\{(I - KC)\bar{e}\bar{e}^T(I - KC)^T\} + E\{K\bar{w}_2\bar{w}_2^TK^T\}, \]  

(2.18)

or,

\[ P = (I - KC)P^-(I - KC)^T + KRK^T. \]  

(2.19)

Expanding,

\[ P = (P^* - KCP^* - P^*C^TK^T + KCP^*C^TK^T) + (KRK^T). \]  

(2.20)

Because the covariance matrix is symmetric, it is equal to its transpose. The third term can be rewritten, which provides an error covariance matrix,

\[ P = P^* - KCP^* - (KCP^*)^T + K(CP^*C^T)K^T + K(R)K^T. \]  

(2.21)

This represents the covariance of the state estimate error regardless of the observer gain
chosen. It is desirable to minimize the state error, so the trace of the covariance matrix must be minimized.

Using the matrix differentiation formulae,

\[
\frac{d \left( \text{trace } FG \right)}{dF} = G^T, \\
\frac{d \left( \text{trace } (FG)^T \right)}{dF} = G^T,
\]

and,

\[
\frac{d \left( \text{trace } FG^T \right)}{dF} = 2FG.
\]

the derivative of the trace of the covariance matrix with respect to the observer gain matrix,

\[
\frac{d(\text{trace } P)}{dK} = -2 \left( CP^{-T} \right)^T + 2K \left( CP^{-T} \right) + 2K \left( R \right)
\]

is determined.

Setting the result of (2.25) to zero provides a relationship defining the point at which the trace of the covariance matrix is a minimum,

\[
P^{-T}C^T = K \left( CP^{-T}C + R \right)\]

(2.26)
Rearranging, the observer gain,

\[ K = P^{-1}C^T \left( CP^{-1}C^T + R \right)^{-1}, \quad (2.27) \]

is found. This is the optimal observer or Kalman Gain Matrix.

In a recursive state estimator, state estimate projections,

\[ \hat{x}_{k+1}^- = A_k \hat{x}_k, \quad (2.28) \]

are based on the current state estimate and state transition matrix. Error projections,

\[ \bar{e}^- = \bar{x} - \hat{x}^- \],

\[ (2.29) \]

can be used to determine an error covariance projection. Incorporating (2.5), and (2.28),

\[ \bar{e}_{k+1}^- = A_k \bar{x}_k + \bar{w}_{1,k+1} - A_k \hat{x}_k, \quad (2.30) \]

gathering terms,

\[ \bar{e}_{k+1}^- = A_k (\bar{x}_k - \hat{x}_k) + \bar{w}_{1,k+1}, \quad (2.31) \]

and using (2.11),

\[ \bar{e}_{k+1}^- = A_k \bar{e}^- + \bar{w}_{1,k+1}, \quad (2.32) \]

a representation of the error projection vector is determined. This function can be used to determine an error covariance projection,

\[ P^- = E\{\bar{e}^- \bar{e}^-^T\}. \quad (2.33) \]
Note that the first term on the right side in (2.32) includes the process noise and sensor noise from the current step, $k$, while the second term includes the process noise that will be present in the next step, $k + 1$. If there is no correlation between the process noise from one step to the next, the expected values of the terms in (2.32) are separable. Thus,

$$P_{k+1}^- = E \left\{ A \bar{e} \bar{e}^T A^T \right\}_k + E \left\{ \bar{w}_1 \bar{w}_1^T \right\}_{k+1},$$

(2.34)

or,

$$P_{k+1}^- = A_k P_k A_k^T + Q_{k+1}.$$  

(2.35)

Equations (2.10), (2.19), (2.27), (2.28), and (2.35) are used in the recursive Kalman Filter.

The recursion algorithm is shown graphically in Figure 2.6.
In the recursive Kalman Filter algorithm, state estimates determined in the prior time step are improved by comparing sensor measurements with those expected from the system model. The mean square state estimate error is computed, then estimates of the expected error and states are determined for the following time step.

2.5 Applied Kalman Filtering, an Example

To demonstrate the use of a Discrete Kalman Filter, consider the interesting problem of fusing data generated by a GPS receiver with inertial navigation systems (INS). INS instrumentation is based on the measurement of linear and rotational acceleration. These data can be integrated to determine position and velocity which are generally used for navigation purposes. The problem with INS instrumentation is that the error present in the sensor data is also integrated, which results in errors in position and velocity which
grow over time. The error associated with GPS-derived measurements of position do not grow over time, but may not be as accurate as necessary for a particular application. The fusion of data from both sources using a Kalman Filter results in an optimal determination of position and velocity.

Position and velocity data from GPS have error characteristics that are correlated in time. This property violates the condition stipulated in (2.34). This is often the case in practical implementations of the Kalman Filter. Mathematically, this means that the output from the Kalman Filter will be sub-optimal, but nonetheless represents a significant improvement in the measurement available from either GPS or INS operating alone.

To construct the state vector and the state transition matrix, it is necessary to have an idea of the dynamic range of the vehicle being navigated. Where an automobile may experience acceleration of less than eight metres per second squared and the driver may be capable of making control adjustments resulting in jerk of less than fifty metres per second cubed, a high-performance jet fighter may be capable of acceleration in excess of fifty metres per second squared and jerk in excess of one hundred metres per second cubed. In the latter instance, it may be necessary to consider higher order dynamics. For this example, it is assumed that higher order dynamics are negligible. The kinematic equation relating position, \( s \), velocity, \( v \), acceleration, \( a \), and jerk, \( j \), from one time step to the next,
is governed by the zero-mean random variable, $q$. This means that jerk is a random process. In the implementation of this filter, the variance of this process noise must be chosen. In practice, assuming jerk to be a random process will be an approximation of reality. Arriving at a model to describe the system dynamics is, in the opinion of the author, the most important aspect in the design of a Kalman Filter. In this example, whether or not the system dynamics can be described by assuming jerk to be a random process is pivotal.

The measurements from a GPS receiver and from an INS,

$$
\bar{y} = \begin{bmatrix} s \\ a \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} r_1 \\ r_2 \end{bmatrix},
$$

(2.37)

also include sensor noise represented by $r_1$ and $r_2$. As with process noise, the variance of the sensor noise must be known or approximated. Based on these simple equations and the identification of process noise and sensor noise variances, the implementation of the Kalman Filter is carried out through the use of the Discrete Kalman Filter equations developed earlier.
In the application described, the position, velocity, and acceleration of the vehicle is overdefined. Either sensor alone could be used to sub-optimally identify all states. The Kalman Filter essentially treats the noise variances as weighting factors to determine an improved estimate of the state variables.
Chapter 3 - Objectives

3.1 Introduction

The successful development of a system that could reduce the frequency of air transportation accidents where runway friction coefficient or limited length of runway is a significant problem would be a major achievement. In the development of such a system, a number of major studies would have to be undertaken. These could include computer modelling of the dynamics of aircraft pertaining to both flight and ground operations, and the on-board instantaneous measurement of both the runway friction coefficient and the length of runway remaining. The incorporation of this and other information such as aircraft mass, measurements of inertia, and wind direction and speed, could lead in time to the development of a safe stopping distance warning system.

The specific purpose of the proposed research was to investigate the feasibility of using an observer system during the roll and takeoff phase of aircraft operation to provide to the pilot the information that is needed to manoeuvre safely. If feasible, such an observer system could be later incorporated within a takeoff performance monitoring system.

While previous work in this field focussed on the design of a takeoff performance monitoring system for use on dry, paved runways, this project examined the factors pertaining to gravel runways where reverse thrust, rather than braking, is the primary
means of deceleration.

3.2  Project Objectives

The objectives of this thesis project were:

1. to investigate, with the aid of a theoretical model, the relative importance of the various parameters influencing the estimation of stopping distance for a specific aircraft type, the British Aerospace Jetstream 31, through installation and flight testing in a typical aircraft, and;

2. to explore the sensing technologies that would be required to measure the required parameters, with special consideration for the application of the Global Positioning System in determination of aircraft acceleration, velocity, and position with respect to the end of the runway.
Chapter 4 - Theoretical Prototype System

4.1 Background

Assuming that all necessary quantities can be measured, a takeoff performance monitor would require a method to project how the speed, position, and acceleration of an aircraft might change in the future based on measurements taken in the past. This necessitates the availability of a mathematical model describing how these parameters vary with respect to one another.

4.2 Parametric Model of an Aircraft During Takeoff

In the construction of such a model, each parameter need not be independently measurable, so long as the system can be observed.

The force of drag on an aircraft,

\[ D = D_3 v_{ia}^2, \quad (4.1) \]

where: \( D_3 \) is a constant parameter for a given aircraft geometry, and;

\( v_{ia} \) is the speed of the aircraft relative to the air.

Applying the convention that a headwind is positive while aircraft speed is positive forward,
\[ D = D_3 (v_a + v)^2. \quad (4.2) \]

Expanding,

\[ D = D_3 v_a^2 + 2D_3 v_a v + D_3 v^2, \quad (4.3) \]

or,

\[ D = D_1 + D_2 v + D_3 v^2, \quad (4.4) \]

where: \( D_a \) are constant parameters for a given aircraft geometry,

\( v_a \) is the component of wind in the direction of the runway, and;

\( v \) is the speed of the aircraft relative to the ground.

Similarly, thrust,

\[ T = T_0 + T_3 v_a^2, \quad (4.5) \]

where: \( T_0 \) is a parameter representing the throttle setting, and;

\( T_3 \) is a parameter to account for increased thrust at higher engine inlet pressures.

As in the derivation for drag,

\[ T = T_0 + T_3 (v_a + v)^2. \quad (4.6) \]

Expanding,
\[ T = T_0 + T_3 v_a^2 + 2T_3 v_a v + T_3 v^2, \quad (4.7) \]

or,

\[ T = T_1 + T_2 v + T_3 v^2, \]

where: \( T_n \) are constant parameters for a given throttle setting.

Simple relationships exist for viscous friction,

\[ F = F_2 v, \quad (4.8) \]

and for the component of weight in the direction of motion,

\[ W = W_1, \quad (4.9) \]

where: \( F_2 \) and \( W_1 \) are constants provided that the runway slope is constant.

Grouping similar parameters and applying Newton’s Second Law,

\[ a = P_1 + P_2 v + P_3 v^2, \quad (4.10) \]

where: \( P_n \) are parameters representing the net force per unit mass acting on the aircraft,

and;

\( a \) is the acceleration of the aircraft in the direction of motion.
4.3 Projection of Displacement

To use this model for the prediction of later displacement requires an equation describing the displacement as a function of speed. From fundamental kinematics,

\[ v = \frac{ds}{dt}, \quad \text{and;} \]

\[ a = \frac{dv}{dt}, \quad (4.12) \]

which can be solved to describe the differential time,

\[ dt = \frac{ds}{v} = \frac{dv}{a}. \quad (4.13) \]

Rearranging, an equation describing the differential displacement,

\[ ds = \frac{v \, dv}{a}, \quad (4.14) \]

is found.

Incorporating the model,

\[ ds = \frac{v \, dv}{P_1 + P_2 v + P_3 v^2}. \quad (4.15) \]

If the instantaneous position and speed are known, the displacement at a reference speed can be determined through integration. The displacement,
\[ s_2 - s_1 = \int_{v_1}^{v_2} \frac{v \, dv}{P_1 + P_2 v + P_3 v^2}, \quad (4.16) \]

where: \( s_1 \) is the instantaneous position;

\( s_2 \) is the predicted position at the reference speed;

\( v_1 \) is the instantaneous speed, and;

\( v_2 \) is the reference speed.

The solution to this integral,

\[ s_2 - s_1 = \frac{c \ln |v + c| - d \ln |v + d|}{P_3 (c - d)} \bigg|_{v_1}^{v_2}, \quad (4.17) \]

where:

\[ c = \frac{P_2 + \sqrt{P_2^2 - 4P_1P_3}}{2P_3}, \quad (4.18) \]

and:

\[ d = \frac{P_2 - \sqrt{P_2^2 - 4P_1P_3}}{2P_3}, \quad (4.19) \]

can be used to conduct an uncertainty analysis for the measured quantities in the model.

### 4.4 Signal Processing Technique

Customarily, the states in a Kalman Filter are time derivatives of one another. This stems from the rigidity of the continuous Kalman Filter, which requires that all states be related
to one another through differentiation in a homogeneous domain. The discrete Kalman Filter is not limited in this way.

### 4.4.1 Standard Treatment of the Discrete Kalman Filter

Based on the dynamics pertaining to the particular application, the designer typically chooses a high derivative to identify as a random process. The lower states are then dependent on the random variable. Each state may also be assigned some random variability to uncouple neighbouring states. For instance, it would not be uncommon to describe the dynamics of an aircraft during its takeoff roll based on its position, \( s \), speed, \( v \), acceleration, \( a \), and jerk, \( j \),

\[
\begin{bmatrix}
    s \\
v \\
a \\
j_{k+1}
\end{bmatrix} =
\begin{bmatrix}
    1 & dt & 0 & 0 \\
    0 & 1 & dt & 0 \\
    0 & 0 & 1 & dt \\
    0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    s \\
v \\
a \\
j_k
\end{bmatrix} +
\begin{bmatrix}
    0 \\
    0 \\
    0 \\
    q_j
\end{bmatrix},
\]  

(4.20)

where: \( dt \) is the difference in time between \( k \) and \( k+1 \), and;

\( q_j \) is a zero-mean random variable.

Such a filter functions best when jerk most closely resembles a zero-mean random process, though this is usually an approximation of reality. For small time steps, it may be considered a reasonable approximation.
4.4.2 Non-linear Manipulation of the Discrete Kalman Filter

In the model,

\[ a = P_1 + P_2 v + P_3 v^2 , \tag{4.21} \]

jerk can be found through differentiation,

\[ j = \frac{d a}{d t} = P_2 a + 2P_3 v a , \tag{4.22} \]

or,

\[ j = P_1 P_2 + \left( P_2^2 + 2P_1 P_3 \right) v + 3P_2 P_3 v^2 + 2P_3^2 v^3 , \tag{4.23} \]

and is clearly not a zero-mean process. The higher derivatives are also non-zero.

On the other hand, velocity derivatives of the model,

\[ \frac{d a}{d v} = P_2 + 2P_3 v , \tag{4.24} \]

\[ \frac{d^2 a}{d v^2} = 2P_3 , \text{ and;} \tag{4.25} \]

\[ \frac{d^3 a}{d v^3} = 0 , \tag{4.26} \]

provide an alternative method of observer construction. The third velocity derivative of acceleration is a zero-mean process. Without approximation, this can be considered a
random process.

Based on this knowledge, a novel observer built with the state transition equation,

\[
\begin{bmatrix}
  s \\
  v \\
  a \\
  \frac{da}{dv} \\
  \frac{d^2a}{dv^2}
\end{bmatrix}_{k+1} =
\begin{bmatrix}
  1 & dt & 0 & 0 & 0 \\
  0 & 1 & dt & 0 & 0 \\
  0 & 0 & 1 & dv & 0 \\
  0 & 0 & 0 & 1 & dv \\
  0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  s \\
  v \\
  a \\
  \frac{da}{dv} \\
  \frac{d^2a}{dv^2}
\end{bmatrix}_k +
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  q_4 \\
  q_5
\end{bmatrix},
\]

was constructed. This is a model for a Kalman Filter that is capable of an optimal estimation despite reference to a non-linear model. This treatment cannot be examined in the continuous Kalman Filter.

4.5 Required Accuracy

In a functional takeoff performance monitor, the device would project the displacement that would occur between the instantaneous speed and a decision speed, \( V_1 \). This displacement would be added to the projected displacement that would occur when decelerating from \( V_1 \) to rest. The total displacement would be compared to the instantaneous measurement of the remaining runway length, and the difference would be displayed to the pilot as a margin of safety in units of distance. There are several factors that could affect the actual margin of safety, most notably the reaction time of the pilot.

Assuming that the pilot would compensate for reaction time by selecting a comfortable margin of safety, the required accuracy of the margin measurement must be selected. For
larger aircraft, a larger margin would be selected. It is therefore appropriate to establish required accuracy based on the length of the aircraft. In the extreme case, a takeoff rejection is initiated at $V_1$ and the pilot has selected a margin of safety that would be completely consumed by the displacement of the aircraft during the reaction time of the pilot. This is explained further in the following chapter. In the instance where no margin of safety remains, it is desirable that the runway remaining when the aircraft has come to rest is no less than one aircraft length. The author has therefore selected the length of the aircraft, measured from nose to tail, as the required accuracy in the measurement of projected displacement. The aircraft used in this experimental investigation measured fifteen metres from nose to tail.

4.6 Uncertainty Analysis

To assess the sensitivity of the theoretical model to uncertainties in the measured quantities, an uncertainty analysis was conducted.

The partial derivatives of displacement with respect to the measured quantities,

$$\frac{\partial (s_2 - s_1)}{\partial v_1} = \frac{v_2 - v_1}{P_1 + P_2v_1 + P_3v_1^2}.$$ 

(4.28)
\[
\frac{\partial (s_2 - s_1)}{\partial P_1} = \frac{\partial c}{\partial P_1} \left[ \ln \left( \frac{v_2 + c}{v_1 + c} \right) + \frac{c(v_1 - v_2)}{(v_1 + c)(v_2 + c)} \right] - \frac{\partial d}{\partial P_1} \left[ \ln \left( \frac{v_2 + d}{v_1 + d} \right) + \frac{d(v_1 - v_2)}{(v_1 + d)(v_2 + d)} \right] + \left( \frac{\partial d}{\partial P_1} - \frac{\partial c}{\partial P_1} \right) \left[ c \ln \left( \frac{v_2 + c}{v_1 + c} \right) - d \ln \left( \frac{v_2 + d}{v_1 + d} \right) \right]
\]

\hspace{1cm}, \hspace{1cm} (4.29)
\[
\frac{\partial (s_2 - s_1)}{\partial P_2} = \frac{\partial c}{\partial P_2} \left[ \ln \frac{v_2 + c}{v_1 + c} + \frac{c(v_1 - v_2)}{(v_1 + c)(v_2 + c)} \right] \frac{P_3(c - d)}{P_3(c - d)}
\]

\[- \frac{\partial d}{\partial P_2} \left[ \ln \frac{v_2 + d}{v_1 + d} + \frac{d(v_1 - v_2)}{(v_1 + d)(v_2 + d)} \right] \frac{P_3(c - d)}{P_3(c - d)} , \quad \text{and;} \quad (4.30)
\]

\[
\begin{align*}
+ \left( \frac{\partial d}{\partial P_2} - \frac{\partial c}{\partial P_2} \right) & \left[ \frac{c \ln \frac{v_2 + c}{v_1 + c} - d \ln \frac{v_2 + d}{v_1 + d}}{P_3(c - d)^2} \right] \\
\end{align*}
\]
\[
\frac{\partial (s_2 - s_1)}{\partial P_3} = \frac{\partial c}{\partial P_3} \left[ \ln \frac{v_2 + c}{v_1 + c} + \frac{c(v_1 - v_2)}{(v_1 + c)(v_2 + c)} \right] - \frac{\partial d}{\partial P_3} \left[ \ln \frac{v_2 + d}{v_1 + d} + \frac{d(v_1 - v_2)}{(v_1 + d)(v_2 + d)} \right] P_3 (c - d)^2
\]

\[
+ \left( \frac{\partial d}{\partial P_3} - \frac{\partial c}{\partial P_3} \right) \left[ c \ln \frac{v_2 + c}{v_1 + c} - d \ln \frac{v_2 + d}{v_1 + d} \right] P_3 (c - d)^2
\]

\[
- \left[ c \ln \frac{v_2 + c}{v_1 + c} - d \ln \frac{v_2 + d}{v_1 + d} \right] P_3^2 (c - d)
\]
together with partial derivatives,

$$\frac{\partial c}{\partial P_1} = -\left( P_2^2 - 4P_1P_3 \right)^{\frac{1}{2}}, \quad (4.32)$$

$$\frac{\partial c}{\partial P_2} = \frac{1 + P_2 \left( P_2^2 - 4P_1P_3 \right)^{\frac{1}{2}}}{2P_3}, \quad (4.33)$$

$$\frac{\partial c}{\partial P_3} = -\frac{P_2 - \sqrt{P_2^2 - 4P_1P_3} - 2P_1P_3 \left( P_2^2 - 4P_1P_3 \right)^{\frac{1}{2}}}{2P_3^2}, \quad (4.34)$$

$$\frac{\partial d}{\partial P_1} = \left( P_2^2 - 4P_1P_3 \right)^{\frac{1}{2}}, \quad (4.35)$$

$$\frac{\partial d}{\partial P_2} = \frac{1 - P_2 \left( P_2^2 - 4P_1P_3 \right)^{\frac{1}{2}}}{2P_3}, \quad \text{and;} \quad (4.36)$$

$$\frac{\partial d}{\partial P_3} = -\frac{P_2 + \sqrt{P_2^2 - 4P_1P_3} + 2P_1P_3 \left( P_2^2 - 4P_1P_3 \right)^{\frac{1}{2}}}{2P_3^2}, \quad (4.37)$$

form the basis of the uncertainty analysis.

To establish approximate parameter values for use in the uncertainty analysis, an empirical analysis can be conducted. From (4.10), $P_3$ is a function of $T_3$, $D_3$, and the mass of the
aircraft. As a conservative approximation, the contribution of $T_3$ can be neglected. The drag force acting on an object,

$$D = 0.22A\rho_{\text{air}}v_{\text{air}}^2,$$  \hspace{1cm} (4.38)

where: $A$ is the frontal area of the object;

$\rho_{\text{air}}$ is the density of air, and;

$v_{\text{air}}$ is the velocity of the object relative to the air.

The frontal area of the Jetstream 31 aircraft is 14.3 metres squared. The density of air is 1.225 kilograms per metre cubed. The maximum allowable takeoff mass of the aircraft is 7000 kilograms. Neglecting $T_3$,

$$P_3 \approx -\frac{D}{mv_{\text{air}}^2} = -0.00055 \text{ [m}^{-1}] .$$ \hspace{1cm} (4.39)

Similarly, a conservative approximation of $P_2$ can be determined by neglecting the variability of thrust and the contribution of viscous friction in the aircraft tires. This requires an approximation of a typical yet conservatively large value for the wind speed, $v_a$. If this value for wind speed is chosen to be equal to fifteen metres per second,

$$P_2 \approx 2P_3v_a = -0.0165 \text{ [s}^{-1}] .$$ \hspace{1cm} (4.40)
Finally, a value for initial thrust, must be chosen. This value would depend totally on the length of the runway. Choosing a typical value for initial thrust,

\[ P_1 = 3.0 \text{[m/s}^2 \text{]} \] \hspace{1cm} (4.41)

Based on these conservative parameter values, partial derivative values,

\[ \frac{\partial c}{\partial P_1} = -1.21 \times 10^1 \text{[s]}, \] \hspace{1cm} (4.42)

\[ \frac{\partial d}{\partial P_1} = 1.21 \times 10^1 \text{[s]}, \] \hspace{1cm} (4.43)

\[ \frac{\partial c}{\partial P_2} = -7.28 \times 10^2 \text{[m]}, \] \hspace{1cm} (4.44)

\[ \frac{\partial d}{\partial P_2} = -1.09 \times 10^3 \text{[m]}, \] \hspace{1cm} (4.45)

\[ \frac{\partial c}{\partial P_3} = -4.40 \times 10^4 \text{[m}^2/\text{s]}, \text{and;} \] \hspace{1cm} (4.46)

\[ \frac{\partial d}{\partial P_3} = -9.85 \times 10^5 \text{[m}^2/\text{s]}, \] \hspace{1cm} (4.47)

are obtained.

Suppose the aircraft in question has an instantaneous speed of thirty metres per second,
and the reference speed is fifty metres per second. Evaluating (4.28) through (4.31), these quantities result in partial derivatives of displacement with respect to measured quantities,

$$\frac{\partial (s_2 - s_1)}{\partial v_1} = \frac{2.00 \times 10^1}{2.01} = 1.00 \times 10^1 [s], \quad (4.48)$$

$$\frac{\partial (s_2 - s_1)}{\partial P_1} = 37.4 \frac{\partial c}{\partial P_1} - 4.65 \frac{\partial d}{\partial P_1} = -5.07 \times 10^2 [s^2]. \quad (4.49)$$

$$\frac{\partial (s_2 - s_1)}{\partial P_2} = 37.4 \frac{\partial c}{\partial P_2} - 4.65 \frac{\partial d}{\partial P_2} = -2.22 \times 10^4 [s \cdot m], \quad \text{and}; \quad (4.50)$$

$$\frac{\partial (s_2 - s_1)}{\partial P_3} = 37.4 \frac{\partial c}{\partial P_3} - 4.65 \frac{\partial d}{\partial P_3} + 1.12 \times 10^6 = -9.83 \times 10^5 [m^2]. \quad (4.51)$$

The equations can be used to determine the effect of errors in the measured quantities.

The manufacturer specified maximum speed error is 0.20 metres per second. As a result of (4.48), there will exist two metres of error in the projected displacement due to this speed error at the conditions outlined. This error will decrease as the reference speed is approached.

In a conventional uncertainty analysis, each measured quantity is treated independently. However, in this particular case, there exists only one real measurement, that of speed provided by the GPS receiver. The Kalman Filter described earlier observes the parameters based on a measurement of speed. As a result, any error in a single parameter will result in corresponding errors in the remaining parameters.
The worst case scenario corresponds to a sudden change in drag coefficient or wind speed that would affect one or both of the parameters $P_2$ and $P_3$ more quickly than the filter can respond. Figure 4.1 shows the resulting error in the projection of displacement that would occur as the aircraft accelerates from an instantaneous speed of thirty metres per second to a reference speed of fifty metres per second as a function of percent error in the parameters $P_2$ and $P_3$.

![Error in Projection of Displacement as a Function of Error in P2 and P3](image)

**Figure 4.1** Error in Projection of Displacement as a Function of Error in P2 and P3

Figure 4.2 shows the error in projected displacement as a function of speed resulting from
10% error in both $P_2$ and $P_3$. Naturally, as the reference speed is approached, the error in the projection of the displacement that will have occurred at the reference speed approaches zero.

**Figure 4.2** Error in Projected Displacement as a Function of Speed

In most cases, the actual error in the quantities determined by the Kalman Filter should be much less than what has been described here. Note, however, that if separate measurement systems were used to directly measure each quantity, the uncertainty in each measurement would result in a projected displacement error orders of magnitude higher. This is a clear advantage of the signal processing technique earlier described.
Chapter 5 - Takeoff Rejection Simulation using a Theoretical Model

Using the theoretical model developed in the previous chapter, a simulation routine was developed. The purpose of this simulation was to demonstrate a simple interface that could provide information to a pilot regarding the margin of error associated with a takeoff rejection at or near $V_1$. Figure 5.1 shows the result of the simulation routine prior to the simulated rejection was initiated.

**Figure 5.1** Projected Displacement and Margin of Safety
The routine estimated a margin of safety of 380 metres. If the rejection occurs prior to reaching $V_1$, a margin of safety of more than 380 metres would have existed. If the rejection occurred at or near $V_1$, some of the margin of safety would have been consumed in the time required for the pilot to react. Figure 5.2 depicts a simulated takeoff rejection initiated just prior to reaching $V_1$. A fraction of a second later the pilot reduced thrust to idle, but some of the margin of safety had already been consumed. A couple of seconds later, reverse thrust was applied. By this time, the margin of safety had decreased to 115 metres.

**Figure 5.2** Margin of Safety during the Response to a Takeoff Rejection
In the simulation routine, the availability of braking is not considered in the projection of displacement. Simulated braking is available, in the event that the pilot requires additional deceleration to improve the margin of safety after the takeoff has been rejected. The simulation is available in the CD-ROM appendix under the “simulation” directory. Note that, to completely reject a simulated takeoff, it is necessary to first call “reject” by clicking the button marked “REJECT”, then set the throttle to idle by clicking the button marked “IDLE”, then reverse thrust by clicking the button marked “REVERSE”.

Optionally, the “BRAKES OFF / BRAKES” button may be toggled to increase the margin of safety. These steps have been included to simulate the required reaction mechanism.

The simulation routine allowed the author to examine the ability to project displacement with known parameters, and to evaluate whether the Kalman Filter introduced in the previous chapter could, in fact, converge to the parameters in the presence of sensor and process noise. The Kalman Filter was also shown to be able to converge to the parameters and thus became a candidate signal processing technique for experimental data.
Chapter 6 - Sensor Selection

6.1 Background

At a minimum, the sensors in a takeoff performance monitor must be able to measure acceleration, speed, and position relative to a known location. This information could be used, with reference to a theoretical model, to predict how the aircraft will behave in the future. Accelerometers have been typically used to measure acceleration. However, because accelerometers do not measure purely acceleration, some method of determining the orientation of the accelerometer with respect to the gravity vector has been required. Typically, this required the use of a gyroscope, which measured the rate of change of orientation. With time, the accuracy of a gyroscope-derived determination of orientation becomes inaccurate because small errors in the raw measurement are integrated over time to determine orientation.

The measurement from an airspeed indicator together with a recently acquired measurement of wind speed has been used to determine aircraft speed, with shortcomings in accuracy due mostly to the time varying nature of wind and the associated delay in obtaining updated measurements. Alternatively, a time-integrated measurement of acceleration could be used. This leads to an accumulation of error over time, requiring re-initialization.
With the recent widespread availability of highly accurate position measurements from the Global Positioning System, a GPS receiver was chosen as a sensor that would provide information regarding the position of the aircraft relative to the end of the runway. A GPS receiver can also be used to measure speed relative to the ground. Of course, during a takeoff roll, speed changes. If a GPS receiver can also be used to measure acceleration with accuracy comparable to accelerometers, there is the possibility that one sensor can be used as the sole source of kinematic information for a takeoff performance monitor.

It may be necessary to include instrumentation capable of measuring the forces present in the landing gear. Such forces could reveal the magnitude of the instantaneous rolling friction, the weight of the aircraft, and the instantaneous normal force. It was decided that the need for such additional sensors would be governed by experimental results. Whether such instrumentation would be necessary depends on the sensitivity of the system to changes in these parameters.

6.2 Acceleration From GPS

The notion of acquiring a measurement of acceleration from GPS is not new. When compared to the measurement obtained from an accelerometer, a GPS-derived measurement of acceleration can be used to determine the gravity vector. This technique has been used in airborne gravimetry to determine the gravitational constant with accuracies on the order of $10^{-5}$ metres per second squared, but requires a substantial amount of data to filter out vibrational disturbances. More recently, it has been proposed that a GPS-derived measurement of acceleration together with an accelerometer could
yield a representation of the gravity vector\textsuperscript{19} to be used as an attitude reference. Such an application would require a real time GPS-derived measurement of acceleration if used on vehicles with rapidly changing attitude.

In aircraft landing and takeoff performance monitoring, the desired acceleration measurement should reflect the overall vehicular acceleration as opposed to vibration of sub-components. Accelerometers are well suited to measurement of vibration, where the influence of gravity need not be removed from the measurement, but a GPS-based measurement is clearly superior in stable, slowly changing vehicular acceleration.

Although accelerometers have been historically used to determine aircraft acceleration, additional instrumentation is required to accurately measure aircraft attitude to remove the significant and adverse influence of the gravity vector. Accelerometers do not respond only to acceleration, but rather the force per unit mass on an element of known mass. The accelerometer measurement is therefore a combination of the components of gravity and acceleration in the direction of the sensing axis of the accelerometer. With reference to Figure 6.1, the accelerometer measurement,

$$a^* = a_x + g_x,$$  \hspace{1cm} (6.1)

where: $a_x$ is the component of acceleration in the accelerometer’s sense-direction, and;

$g_x$ is the component of gravity in the sense-direction of the accelerometer.
Figure 6.1 “Accelerometer Gravity Error”

“Accelerometer Gravity Error” results from an accelerometer measuring a component of the gravity vector that cannot be determined without an accurate measurement of the sensor inclination with respect to horizontal. Measurement of inclination always requires additional instrumentation. Depending on the sensor used to measure inclination, the accuracy of the measurement usually degrades over time.

The component of gravity in the sense-direction,

\[ g_x = g \sin \theta. \]  \hspace{1cm} (6.2)

where: \( \theta \) is the angle between the sense-direction and horizontal.

Thus, for small angles the gravity vector introduces an error of 0.171 m/s\(^2\) per degree of inclination. This problem is avoided through the use of a GPS-derived measurement. The selection of a GPS receiver as the primary source of kinematic information is an especially appropriate choice given the need to determine the location of the aircraft with respect to the end of the runway, an application in which a GPS receiver is well employed.
6.3 Required Accuracy

The level of uncertainty in a GPS-derived measurement of acceleration depends on the
type of filter used to remove noise that is amplified by differentiation. In any case, the
amount of error will not depend on the magnitude of the instantaneous signal. Rather, the
standard deviation of noise depends primarily on the number of satellites in view.

Assuming constant deceleration, the instantaneous stopping distance of a vehicle,

\[ s = \frac{v^2}{2a}, \]  \hspace{1cm} (6.3)

where: \( v \) is the instantaneous speed of the vehicle, and;

\( a \) is the magnitude of acceleration of the vehicle.

The sensitivity of the stopping distance,

\[ \Delta s = \Delta v \frac{\partial s}{\partial v} + \Delta a \frac{\partial s}{\partial a}, \]  \hspace{1cm} (6.4)

where: \( \Delta v \) is the uncertainty in the measurement of speed, and;

\( \Delta a \) is the uncertainty in the measurement of acceleration.

Substituting expressions for partial derivatives,

\[ \Delta s = \Delta v \frac{v}{a} - \Delta a \frac{v^2}{2a^2}. \]  \hspace{1cm} (6.5)
Rearranging (6.3) to solve for deceleration, substituting into (6.5), and solving for deceleration uncertainty gives:

$$\Delta a = \Delta v \cdot \frac{v}{s} - \Delta s \cdot \frac{v^2}{2s^2}.$$  \hspace{1cm} (6.6)

Instances where deceleration uncertainty has the most impact occur when the forward speed is high and when the rearward acceleration is low. To establish an acceptable level of uncertainty in the measurement of deceleration, consider a modest $V_1$ on a long runway. Runways are seldom longer than 4000 m. Over half of the length of the runway would be required to reach $V_1$, so assume $s = 2000$ metres and $v = V_1 = 90$ metres per second.

The margin of safety in the stopping distance would need to be larger than the length of the aircraft, so the uncertainty in the estimation of stopping distance can be conservatively chosen as $\Delta s = 150$ metres. From GPS, a typical speed uncertainty determined using constant speed trials, as described shortly, was $\Delta v = 1.12$ metres per second. Note that at the time of this preliminary experimental investigation, intentional degradation was present in the signal used by the GPS receiver.

Substituting these values into equation (6.6) yields a conservative maximum acceptable uncertainty in the measurement of deceleration:

$$\Delta a = 0.101 \text{ m/s}^2.$$ \hspace{1cm} (6.7)
While a lower speed may apply to many aircraft, the runway length should be considered very conservative. In many cases, higher uncertainty may be acceptable. This analysis was intended to provide a nominal uncertainty to which results could be compared.

6.4 Experimental Investigation

An experimental investigation was conducted to determine the accuracy with which a GPS receiver could measure acceleration. Note that, at the time of this investigation, SA was present in the GPS signal structure resulting in position errors of approximately 100 metres. Although position errors existed, speed errors were very small resulting in negligible acceleration error due to differentiated SA. The GPS-derived measurements collected during this investigation were compared to measurements derived from carefully mounted accelerometers to determine the accuracy with which a GPS receiver could be used to measure acceleration.

A GPS receiver (NovAtel 3151RE) capable of collecting satellite range measurements at a rate of 20 Hz was selected for use in the test apparatus. The receiver logged position and velocity at 10 Hz. The velocity measurement from the GPS receiver in the test apparatus was derived from time differentiation of position or carrier phase Doppler measurements owing to the manufacturer's proprietary algorithm. The acceleration measurement was a filtered time differentiation of this velocity measurement, obtained using a Kalman Filter. In this Kalman Filter, the third derivative of position was considered to be a random process. This is of course untrue, but was a reasonable approximation of the dynamics.
In state-space form, the system dynamics,

\[
\begin{bmatrix}
    s \\
v \\
a \\
\end{bmatrix}_{k+1} =
\begin{bmatrix}
    1 & dt & 0 & 0 \\
    0 & 1 & dt & 0 \\
    0 & 0 & 1 & dt \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    s \\
v \\
a \\
\end{bmatrix}_k + 
\begin{bmatrix}
    0 \\
    0 \\
    0 \\
    w_k
\end{bmatrix},
\]

(6.8)

and the measurements,

\[
y_k = \begin{bmatrix}
    0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    s \\
v \\
a \\
\end{bmatrix}_k + v_k,
\]

(6.9)

where: \(s\), \(v\), \(a\), and \(j\) are position, speed, acceleration, and jerk, respectively, form the foundation of the Kalman Filter. This filter requires identification of the variance of the two random variables. The measurement noise is easily approximated during constant speed trials as later discussed. This leaves one variance, that of the process noise, to be chosen.

Figure 6.2 shows the speed of a vehicle obtained from a GPS receiver in the presence of selective availability, and acceleration computed using the Kalman Filter. This measurement of acceleration can be compared to a measurement obtained from a bank of accelerometers.
Figure 6.2 Speed and Acceleration of a Test Vehicle

Speed and acceleration of a test vehicle were derived from GPS data in the presence of selective availability. The measurement of acceleration required filtering to remove noise due to differentiation.

Testing was undertaken to verify the accuracy of the acceleration measurement derived from GPS data. The apparatus consisted of a vehicle-mounted GPS receiver, a bank of accelerometers (Analog Devices ADXL 202) mounted with parallel axes of measurement, and a data acquisition system. The vehicle was rail-mounted with no suspension system.
Four identical accelerometers were used to provide a confident measure of the acceleration.

The data acquisition system collected these data at a rate of 20 Hz, electrically synchronized with the GPS receiver’s collection of raw satellite signals. The data acquisition system for the accelerometers, which used an analog-to-digital data acquisition computer card, was separate from the data collection system for the GPS receiver, which used serial communications. The GPS receiver was equipped with strobe pins for both input and output triggering. One of these strobes fired on receipt of raw satellite data. This strobe was used to trigger the collection of data from the accelerometers. Another strobe connection allowed the GPS receiver to log the time at which it received an input voltage step. This strobe was used to record the times at which accelerometer data collection began and ended. In this way, the data from both the accelerometers and from the GPS receiver could be synchronized in time.

During constant-speed trials, the accelerometers were used to determine the slope of the rail surface so that the influence of the gravity vector could be calculated. This slope information was cross matched with geographic location through the use of differential GPS. Twenty constant-speed trials were conducted, yielding a reliable measurement of slope. This method of accounting for rail slope implicitly accounted for any bias errors present in the accelerometers. During trials where the vehicle speed varied, the accelerometer data were corrected for the influence of minor pitch changes by subtracting the known slope at the instantaneous position. Both measurements of acceleration were
filtered using the same algorithm. The GPS-derived acceleration measurement was then compared with acceleration data from the bank of accelerometers, after accounting for the effect of gravity. This is shown schematically in Figure 6.3.

![Signal Processing Schematic](Figure 6.3)

6.5 Results and Discussion

The use of redundant accelerometers provided increased confidence in the measurement of acceleration. Measurements from each of the four accelerometers used in the experimental apparatus agreed with one another very well. The covariance matrix describes how the data collected from each sensor varied from one another. In completely uncorrelated data, off diagonal values would be zero. In completely correlated data, all values would be equal. For the collected accelerometer data, a covariance matrix,
$\text{cov} = \begin{bmatrix} 1.779 & 1.776 & 1.775 & 1.772 \\ 1.776 & 1.779 & 1.775 & 1.776 \\ 1.775 & 1.775 & 1.779 & 1.773 \\ 1.772 & 1.776 & 1.773 & 1.779 \end{bmatrix} \text{m}^2/\text{s}^4, \quad (6.10)$

was determined.

It can be concluded that the accelerometer data, while being variable with a standard deviation on the order of 1.33 metres per second squared, were highly correlated. This demonstrated that the accelerometer data represented a confident measure of the acceleration of the vehicle component to which the accelerometers were attached.

Figure 6.4 shows a comparison of the GPS-derived acceleration with that from the accelerometers for one of ten trials where the speed of the vehicle was varied. Other trials yielded similar results. While both measurements of acceleration were filtered in exactly the same manner, there is clearly no superiority in the accelerometer measurement. It had been expected that given the care with which the accelerometer data were corrected, an approach that would not be feasible in practice, the GPS-derived acceleration would be notably time-delayed and would contain noise. Conversely, it would appear that in those instances where the vehicle acceleration was steady and most similar to what might be expected of a large aircraft, such at the period between 25 seconds and 30 seconds, the two measurements were in agreement.
Measurements of the acceleration of a test vehicle derived from GPS data were compared with accelerometer-derived measurements. While the GPS-derived measurement required filtering to remove noise due to differentiation, the accelerometer-derived measurement required filtering to remove measurements of vibration.

Figure 6.5 shows the calculated difference, for the same trial, between the two measurements of acceleration. This does not represent the error in the GPS-derived measurement, as the accelerometer measurement also lags the “real” acceleration because
of filtering. The standard deviation of the difference was 0.054 metres per second squared. This falls well within the established conservative maximum uncertainty of 0.101 metres per second squared. Closer analysis showed that the calculated difference falls within the maximum uncertainty over 90% of the time.

Figure 6.5 Difference between Acceleration Measurements

With regard to the dynamic range of this investigation, it should be noted that the acceleration and speed associated with aircraft takeoff and landing are typically larger than those investigated. In the investigation, speeds in excess of 10 metres per second were
not experienced and acceleration was typically 0.5 metres per second squared. This
difference in dynamic range should have little effect on the accuracy or resolution of the
GPS-derived speed measurement, which is governed by jerk. Because the corresponding
accuracy of the acceleration measurement was dependent on the ratio of speed accuracy to
the change in speed, this accuracy should improve at higher accelerations.

It has been demonstrated that the Global Positioning System is able to provide a
measurement of vehicle speed that is sufficiently reliable to determine acceleration with an
uncertainty of under 0.10 metres per second squared. This accuracy should be achievable
for acceleration in excess of 0.5 metres per second squared. Clear advantages in using a
GPS receiver over the more conventional sensor, an accelerometer, include
insusceptibilities to the gravity vector, vibrational disturbances, and temperature
fluctuations.
Chapter 7 - Model Validation through Experimental Investigation

For the experimental investigation in this phase of the project, it was decided that the results would carry much more credibility if the data collected represented normal operating conditions as opposed to simulated emergency scenarios. A local airline cooperated in the research project. The airline operated four twin turboprop Jetstream 31 aircraft. The airports visited by these aircraft on a daily basis included locations as far north as Uranium City, located at north latitude of 59.56 degrees, and as secluded and underserviced as Wollaston Lake, shown in Figure 7.1.

Figure 7.1 Wollaston Lake Airport
The Jetstream 31, also known as the British Aerospace 3112, is a 19-passenger pressurized aircraft. One of the four aircraft operated by the airline was already equipped with a navigational GPS receiver where the antenna was mounted above the cockpit on the exterior of the aircraft. Because the principle of operation of a GPS receiver requires a straight line view of GPS satellites, such an externally mounted antenna was very useful in collecting data for this experiment, if not an absolute necessity.

A fundamental principle in dealing with a commercial airline was established very early in the project. It was absolutely crucial that the design of the experimental system require minimal modification to existing aircraft systems. Adherence to this principle would not only minimize the time required to install the necessary components, but would also simplify the process of obtaining permission from Transport Canada.

A more detailed examination of the aircraft revealed the existence of a docking station for a processor in a traffic collision avoidance system (TCAS), that had been removed due to serviceability problems. While needed in more populated areas, the use of such a device in Canada is not a regulatory requirement. Upon further examination, it was determined that the TCAS docking station was equipped with a dedicated circuit breaker as well as connections to antennae both on the belly of the aircraft and over the cockpit a short distance behind the antenna for the GPS receiver. The radio frequency used by the TCAS was in the same band as that used by the GPS receiver, and the characteristic attenuation of the coaxial cable between the TCAS docking station and the overhead antenna was similar to GPS receiver requirements. It was decided that a commercially available GPS
antenna signal splitter would be used to share the signal from the existing GPS antenna with the navigational GPS receiver and the project test equipment. Appendix 1, the type certificate application, describes this in further detail.

With the physical and electrical constraints for the test equipment dictated by the existing hardware in the aircraft, a data collection system was designed and constructed. The only modifications required in the aircraft were the insertion of a GPS antenna signal splitter and the reconnection of a TCAS antenna cable.

The design and implementation of test equipment for this phase of the project required considerable care. With the knowledge that the most invasive aspect of the project would be the sharing of a navigational GPS antenna, testing was undertaken to verify that no adverse effects would be experienced. The process of obtaining Transport Canada certification for the installation also required technical expertise, including adherence to airworthiness regulations and conformity testing.

7.1 **Antenna Splitter Evaluation**

A simple test was performed to evaluate the integrity of a GPS antenna signal when split between two GPS receivers. The carrier to noise ratio pertaining to individual satellites was measured by two GPS receivers. A comparison was made between the carrier to noise ratio obtained while employing the signal splitter to that obtained in a single receiver configuration. The experimental apparatus is shown in Figure 7.2.
Figure 7.2 Splitter Evaluation Experimental Apparatus

The equipment used for this test included two GPS receivers (NovAtel OEM2, NovAtel OEM3), a GPS antenna (NovAtel Model 531), an antenna signal splitter (Starlink BT-2DGPS), and the necessary cabling. Antenna power was turned off in the OEM2 receiver using internal jumper settings. One 5 metre TNC male to male RG223/U coaxial cable assembly was used to connect the OEM2 receiver to the secondary GPS connection of the antenna splitter. One 5 metre TNC male to male RG223/U coaxial cable assembly was used to connect the OEM3 receiver to the primary GPS connection of the antenna splitter. Power from this receiver was used to activate the amplification circuitry in the GPS antenna.
The GPS antenna was connected to the splitter using a TNC male to male adaptor. Data pertaining to channel tracking status were collected from the OEM3 receiver. After approximately ten minutes, the splitter was quickly removed from the configuration so that the OEM3 receiver was connected directly to the GPS antenna. Data collection was not interrupted during this period. After approximately ten minutes, the apparatus was returned to its original configuration. Data collection was not interrupted during this period. Data were collected for yet another ten minute period.

7.1.1 Results
The carrier to noise ratio pertaining to three different satellites measured by the primary GPS receiver is shown in Figures 7.3, 7.4, and 7.5. The carrier to noise ratio was not affected by the inclusion of a GPS antenna signal splitter. The antenna signal was provided directly to the primary receiver between Time = 251230 seconds and Time = 251910 seconds. At other times, the signal was split with the use of the antenna splitter.

7.1.2 Interpretation and Discussion
The inclusion of a signal splitter results in a theoretical signal attenuation of 3 dB. The results show that this attenuation is not manifested in the measurement of the carrier to noise ratio. It can be concluded that the noise present in the signal was equally attenuated. The carrier to noise ratio pertaining to individual satellites governs the signal rejection decision in a GPS receiver. As long as the total signal attenuation between the antenna and the receiver does not exceed the manufacturer’s specification, the use of a signal splitter will not adversely affect receiver performance.
Figure 7.3 The carrier to noise ratio of satellite PRN 23
Figure 7.4 The carrier to noise ratio of satellite PRN 17
Figure 7.5 The carrier to noise ratio of satellite PRN 26
7.2 Certification

The certification process involves showing compliance with airworthiness regulations through testing or evaluation. Transport Canada provided a list of regulations to which adherence would have to be demonstrated. As well, an explanation of the purpose of the modification, together with an outline of mechanical and electrical considerations, was required.

The basis of certification for the test aircraft was the United States Federal Aviation Administration’s Federal Airworthiness Regulation Part 23, which applies to commuter aircraft that are “propeller-driven, multiengine airplanes that have a seating configuration, excluding pilot seats, of 19 or less, and a maximum certificated takeoff weight of 19,000 pounds or less.”

The purpose of the modification was to collect position and velocity data during takeoff and landing of a Jetstream 31. These data were to be used to investigate the feasibility of a landing and takeoff performance monitoring system for passenger aircraft that frequently travel to airports with gravel runways in the Far North. The equipment required consisted of a unit containing a portable computer and a GPS receiver (NovAtel OEM2), and an antenna splitter used to acquire the signal from an existing GPS antenna.

The Global Positioning Data Recorder (GPDR) system, shown in Figure 7.7, was designed to be minimally invasive and to take advantage of existing equipment wherever possible. Transwest Airlines (formerly Air Sask Aviation) agreed to carry the equipment
onboard the aircraft registered C-FSEW, a Jetstream 31 equipped with a GPS receiver (Bendix / King KLN 0089B). A GPS antenna splitter was employed to acquire the signal from the existing GPS antenna. The specifications for the GPDR are shown in Table 7-1.

![Global Positioning Data Recorder](image)

**Figure 7.6** Global Positioning Data Recorder

The existing configuration of the aircraft included the hardware for a TCAS (Bendix / King CAS 66) installation. The TCAS processor had been removed due to serviceability problems as it was not a required instrument. It was decided that the tray for the TCAS processor would be used as the station for the data recorder. The TCAS infrastructure was used to supply power to the unit. There was coaxial cable running from the TCAS tray to an unused antenna mounted at station 130, just aft of the cockpit bulkhead as
shown in Figure 7.9. The existing GPS antenna was within a few inches of this location.

<table>
<thead>
<tr>
<th>CHARACTERISTIC</th>
<th>DESCRIPTION</th>
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</thead>
<tbody>
<tr>
<td>Form Factor</td>
<td>4 MCU (1/2 long ATR)</td>
</tr>
<tr>
<td>Overall Dimensions</td>
<td>12.750&quot; x 9.125&quot; x 5.000&quot;</td>
</tr>
<tr>
<td>Weight</td>
<td>11 pounds</td>
</tr>
<tr>
<td>Power Requirements</td>
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</tr>
<tr>
<td>Voltage</td>
<td>Nominal: +28 Vdc</td>
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<tr>
<td></td>
<td>Range: +17 to +40 Vdc</td>
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<tr>
<td>Current</td>
<td>Nominal: 1.8 A</td>
</tr>
<tr>
<td></td>
<td>Maximum Operating: 2.0 A</td>
</tr>
<tr>
<td></td>
<td>Bootup: 2.2 A</td>
</tr>
<tr>
<td>Power</td>
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<tr>
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<tr>
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<tr>
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<td>-30 °C to +60 °C</td>
</tr>
<tr>
<td>Cooling</td>
<td>Convection</td>
</tr>
</tbody>
</table>

7.2.1 Mechanical Considerations

An aluminum enclosure was constructed that contained the portable computer and the GPS receiver. The unit weighed 5.0 kilograms. It was secured in place using hold down pins on the existing TCAS processor tray, located just aft of the rear right side passenger seats, as shown in Figure 7.7.
Figure 7.7 GPDR Installed

The GPS antenna splitter, shown in Figure 7.8, weighed 0.15 kilograms and was attached to the GPS antenna using a TNC male to male elbow adaptor. The coaxial cable supplying the navigational GPS receiver was connected to the primary branch of the splitter using the existing TNC connector. One of four coaxial cables that ran between the TCAS tray and the TCAS directional antenna was connected to the secondary branch of the splitter using the existing TNC connector.

7.2.2 Electrical Considerations

There was a dedicated 5 A circuit breaker for the TCAS tray at the auxiliary avionics circuit breaker panel. This circuit drew power from the 28 VDC main avionics bus. The
record unit drew 1.8 A at 28 VDC and was internally protected with a 3 A normal blow fuse. This level of power consumption was identical to that of the removed TCAS processor.

![Figure 7.8 Antenna Splitter](image)

Figure 7.8 Antenna Splitter

Power was supplied to the GPS antenna internal amplifier circuitry by the existing GPS receiver through the primary branch of the antenna splitter which allowed DC current to pass. The second receiver acquired the antenna signal from the secondary branch. The splitter provided isolation in excess of 20 dB between the primary and secondary branches.
Figure 7.9 Left Side View of Aircraft

The positions of the navigational GPS antenna and the TCAS antenna are shown. The GPDR processor was installed inside the aircraft behind the rear passenger seats as indicated. Numbers along the bottom of the photograph indicate distances, in inches, from the nose of the aircraft. These distances are also known as “stations” for purposes of weight and balance.
7.3 Installation

The navigational GPS antenna was located just forward of the cockpit bulkhead, directly overhead. The overhead cockpit circuit breaker panel was removed to gain access to the antenna. There was one coaxial cable attached to the GPS antenna with a male TNC connector. This cable led to the navigational GPS receiver. The receiver coaxial cable was detached. The TNC male adaptor of the antenna splitter assembly was attached to the GPS antenna. Test C2 was then successfully performed. Test C2 required that, when installed, the combined structure of the antenna splitter, TNC male to male elbow adaptor, and GPS antenna be shown to withstand a force applied vertically down at the centre of the antenna splitter of no less than 7 Newtons without permanent deformation of the structure at any location. The purpose of this test was to show that the new hardware would be able to withstand the maximum certified acceleration that the aircraft might experience. A small test mass was used for this purpose.

The receiver coaxial cable was then attached to the “Primary GPS” connector on the antenna splitter. The TCAS directional antenna was located just aft of the cockpit bulkhead, directly overhead. The overhead cabin panelling was removed to gain access to the antenna. There were four coaxial cables attached to the TCAS directional antenna with male TNC connectors. The forward connector was colour coded yellow. The yellow coaxial cable was detached, drawn forward, then attached to the “Secondary GPS” connector on the antenna splitter. The overhead cabin panelling and the circuit breaker panel were then replaced.
The GDPR processor was installed in the TCAS processor tray just aft of the right side passenger seats. The TCAS processor hold-downs were used to secure the GDPR processor in place. Test C1 was then successfully performed. Test C1 required that, when installed, the GDPR processor be shown to withstand a force applied vertically down of no less than 230 Newtons without permanent deformation of the processor structure or separation from the tray. The purpose of this test was to show that the new hardware would be able to withstand the maximum certified acceleration that the aircraft might experience.

7.4 System Summary

The prototype monitor was certified\(^2\) for use as a Global Positioning Data Recorder (GPDR) and was installed in a 19-passenger British Aerospace 3112 operated by an airline servicing far-northern Canadian airports. The full text of the Supplemental Type Certificate Application including instructions for the installation and testing of the GPDR is available in Appendix 1. The particular aircraft was equipped with a navigational GPS receiver (Bendix King KLN 89B) with a permanent active patch antenna (Bendix King KA 92) installed over the cockpit with a clear view of the sky. A signal splitter (Starlink BT-2DGPS) was installed that allowed the GPS antenna signal to be shared by the navigational GPS receiver and the GPDR. Figure 7.10 depicts the electrical configuration of the complete system. Figure 7.11 shows the internal configuration of the GPDR. The receiver contained in the GPDR logged position and velocity at a rate of 10 Hz to a portable computer that stored the data to a disk drive. The velocity measurement from the GPS receiver in the test apparatus was derived from time differentiation of position or
carrier phase Doppler measurements owing to the manufacturer's proprietary algorithm.

7.5 Data Collection Software

Custom serial data collection software was programmed to extract the necessary information from the GPS receiver in the GPDR. To avoid excess data collection, a technique for recording only data pertaining to takeoff and landing was required. When an aircraft is taxied before takeoff or after landing, speeds can be similar to the first few seconds of a takeoff. The amount of time spent taxiing, however, is nearly one hundred times more than the amount of time spent accelerating for takeoff.

It was necessary to develop logic that would decide based on speed and acceleration whether a takeoff or landing was occurring. Only takeoff and landing information was recorded. The software functioned in a completely autonomous mode, capable of collecting several months of data at a time without human intervention.
Figure 7.10 Global Positioning Data Recorder - System Electrical Schematic
Figure 7.11 Internal Configuration of Global Positioning Data Recorder

Bill of Materials

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<tr>
<th>Part</th>
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<tr>
<td>3.2 - Antenna Adaptors</td>
<td>TNC Male to Male Adaptor / TNC Female to Female BH</td>
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<tr>
<td>3.3 - Power Cable</td>
<td>2 Conductor 18 Gauge Power Cord</td>
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<tr>
<td>3.4 - GPS Receiver</td>
<td>Novatel PWRPAK 3151RE GPS Receiver</td>
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<td>3.5 - Power Control Unit</td>
<td>Power Converter and Relay Logic Circuit</td>
</tr>
<tr>
<td>3.6 - Fuse</td>
<td>3.0 Amp Normal Blow Fuse</td>
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<tr>
<td>3.7 - Interface Patch</td>
<td>4 Conductor 24 Gauge Interface Wiring</td>
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<tr>
<td>3.8 - Serial Patch</td>
<td>9 Conductor 28 Guage Serial Communications Cable</td>
</tr>
<tr>
<td>3.9 - Laptop Computer</td>
<td>Compaq LTE 5000 Pentium 75 Portable Computer</td>
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Drawing Number: SP-A02080600
Global Positioning Data Recorder Processor
Sub-Assembly Schematic

June 8, 2000      Not to Scale
Shane D. Pinder
28 Bell Crescent
Saskatoon, SK S7J 2W3
7.6 Results and Discussion

At the beginning of the project, the test aircraft had been used extensively to visit northern airports. Shortly before certification, the airline merged with another carrier and the aircraft in question was re-tasked to airports with paved runways. While somewhat unfortunate, this did not affect the utility of the data collected during the takeoff phase. The data pertaining to landings, however, could not be modelled using a theoretical model developed for use on gravel runways where braking would not be used.

Between September 15, 2000 and December 12, 2000, 175 takeoffs were recorded. During a typical takeoff, the pilot adjusted control settings until the aircraft reached a speed of twenty-five metres per second. In view of this, the signal processing technique was designed to determine parameters with iterations beginning after the aircraft reached a speed of thirty metres per second. This allowed the dynamics of the aircraft to be reasonably determined with reference to the theoretical model. Once this speed had been reached, the signal processing algorithm continuously projected the displacement that the aircraft would have at an arbitrary speed of fifty metres per second. This speed was chosen as a common reference for all takeoffs as it was always less than the takeoff speed of the aircraft. The projected displacement was then compared with the actual displacement of the aircraft when it reached a speed of fifty metres per second. Figure 7.12 shows the projected displacement as a function of the instantaneous speed for a typical takeoff.
Figure 7.12 Projection of Displacement in a Typical Takeoff.

The projected displacement converged to a value accurate to within ten metres before the aircraft had reached a speed of thirty-five metres per second. The projected displacement error was the amount by which the displacement at a speed of fifty metres per second differed from the instantaneous predicted displacement as a function of speed.
Figure 7.13 shows the results of parameter estimation and stopping distance projection for another typical takeoff. Note that variations in the projected displacement corresponded to variations caused by noisy segments in the acceleration filter. Note also that the noise present in the unfiltered acceleration appears to have been correlated in time. This indicates that either sensor noise or process noise was time correlated. Recall from the discussion of the discrete Kalman Filter in section 2.4 that process noise must be uncorrelated in time for the Kalman Filter to yield an optimal result. There is no such restriction on sensor noise.
Figure 7.14 shows the results of parameter estimation for another typical takeoff. Note that there was a gradual decrease in the estimated value for the parameter, \( P_2 \), corresponding to the convergence of the prediction of displacement to the actual displacement that occurred when the aircraft reached fifty metres per second. Note also that the unfiltered acceleration measurements were again time correlated. Finally, note the lack of variation in the estimation of the value of the parameter, \( P_1 \), which was calculated each iteration after all other parameters were determined. This indicates that the theoretical model was consistent with the measurement data.
Figure 7.15 shows the results of parameter estimation for another typical takeoff. Most notable in this example is the level of noise present in the unfiltered acceleration values. This resulted in slow convergence of the parameter, $P_2$, which in turn resulted in slow convergence of the projection of aircraft displacement. Note that in this far less than ideal situation, the projection of displacement was in error by twenty metres when the aircraft was travelling at thirty metres per second. By the time the aircraft had reached a speed of thirty-seven metres per second, the error in the projection of displacement was less than fifteen metres.
Data were collected over several months. Figure 7.16 shows a scatter plot of all data collected during 175 takeoffs. The solid lines represent the standard deviation of error as a function of speed. Data pertaining to a subset of individual takeoffs are contained in Appendix 2. Data for all takeoffs are contained in the CD-ROM appendix.

![Figure 7.16 Scatter Plot](image)

**Figure 7.16** Scatter Plot

### 7.7 Conclusions

With regard to the parametric model, it was hypothesized that the parameter, $P_3$, would be a characteristic of the aircraft engines and is therefore a parameter that would change
slowly, rendering this parameter a constant for any single takeoff. A filter with an
effective time constant of several takeoffs in length was used to identify this parameter. In
theory, the parameter, $P_2$, combines the effects of runway characteristics, weather
conditions, and wheel bearing friction. A filter with a time constant a few seconds in
length was used to identify this parameter. The convergence of the remaining parameter,
$P_1$, showed that this was an entirely acceptable treatment of the parameters, $P_3$ and $P_2$.

The aircraft used in this experimental investigation measured fifteen metres from nose to
tail. From the data collected, it was concluded that a projection of displacement can be
determined to within an uncertainty of fifteen metres in sufficient time to alert the pilot of
an unsafe situation.
Chapter 8 - Major Conclusions and Recommendations

The feasibility of constructing a takeoff performance monitor that can project the displacement of an aircraft has been demonstrated. Improvements in the theoretical model should result in an improvement of the accuracy of the projection and an associated improvement in the time to convergence of a solution.

8.1 Takeoff Performance Monitoring - Future Work

It has been demonstrated that the development of a TOPMS for use in the far-northern region using a GPS receiver as the sole sensor is technically feasible. To assess the feasibility of large-scale implementation of such a device, the following recommendations are put forward.

8.1.1 Modular Device Development

It is recommended that a prototype TOPMS be constructed for potential integration in candidate test aircraft. The hardware for such a device could be a commercial aviation GPS receiver which would include a display. The integration of TOPMS algorithms could be performed through modification of existing firmware within the GPS receiver. Such a device would include a graphical user interface which would alert the pilot of the instantaneous margin of safety.
8.1.2 Target Environment Testing

Medical ambulance aircraft in the far-northern region encounter marginal conditions more often than commercial airlines. This arises primarily due to the varied airports that are visited by medical ambulance aircraft, and the infrequency with which the same airport is visited. In some cases, the length of the runway would not allow acceleration to \( V_1 \) followed by safe deceleration to rest. It is recommended that medical ambulance aircraft be used in target environment testing. It is expected that the data that could be collected on such a platform would represent a worst case scenario from which refinements to the TOPMS algorithm could be made.

8.2 Device Development - Future Work

There are likely many refinements that could be made to the proposed device configuration that could result in a technically improved TOPMS. For example, a fully integrated commercial INS could easily improve the performance of a TOPMS, but at prohibitively excessive expense. The following recommendations are put forward as areas where cost effective refinements could be most easily realized and where safety improvement is most significantly addressed.

8.2.1 GPS/INS Sensor Integration

The integration of GPS with inertial sensors is by no means new technology. While inertial measurements can provide an accurate determination of acceleration and angular rate, the time integration of these measurements required for positioning results in drift error due to the inevitable time integration of measurement errors. On the other hand,
GPS measurements can provide bounded position and velocity information, but do not respond quickly to platform acceleration. In extreme cases, platform acceleration can cause a GPS receiver to lose signal lock.

The fusion of measurements from GPS and inertial sources results in an improvement over the measurements from either source alone. To date, the successful fusion of GPS with INS measurements has been confined to system level integration. In other words, the output from GPS and the output from an INS are combined in a separate observer.

Since the original design of GPS technology, it has been hypothesized that an improved measurement could be obtained if inertial aiding from an inertial measurement unit (IMU) were used within the tracking loops in the GPS receiver, as shown in figure 8.1.

As described in section 2.2.2, a GPS receiver typically treats any acceleration as an unmodelled disturbance within the tracking loops. As a result, the tracking loops filter away the effects of acceleration and any higher order dynamics. In extreme cases, such as when high acceleration is experienced, this can cause the receiver to lose signal lock because the receiver expects that it is moving at a constant speed. If inertial aiding of the tracking loops were successful, this loss of lock could be avoided.

Inertial aiding of GPS tracking loops would also improve the estimation of parameters within the takeoff performance monitoring algorithm. Like acceleration, higher order dynamics such as the velocity derivatives of acceleration are also filtered away by the
tracking loops in a typical GPS receiver.

**Figure 8.1** Phase Locked Loop with Inertial Aiding

### 8.2.2 Actuator Integration - Automatic Deceleration Systems

In the long term, the successful development of a system that could reduce the frequency of air transportation accidents where runway friction coefficient or limited length of runway is a significant problem would be a major achievement. Such a system may require a fully automated deceleration system to overcome problems associated with pilot reaction time. This would effectively remove the pilot from the control loop at a time when an immediate procedural response is required. The development of such a system would require significant industrial support, which is not currently available. There are, however, other applications where a similar system would be advantageous.
8.3 Other Applications - Unmanned Autonomous Vehicles (UAVs)

UAVs are currently used in military applications for aerial surveillance and, in limited capacity, as an attack vehicle. The Predator UAV is shown in figure 8.2. Aircraft of this sort are piloted remotely and, of course, are flown in hostile environments.

![Predator UAV](image)

**Figure 8.2** Predator UAV

The next generation of military UAVs will extend the attack capability to the extent available from fighter aircraft. Unmanned Combat Air Vehicles (UCAVs), such as Boeing’s X-45A shown in figure 8.3, are similar in size, instrumentation, and cost to a typical fighter aircraft. Consequently, onboard control systems will have the capability to reproduce the control reliability of a human pilot during all phases of flight, including takeoff and landing. This would be the ideal proving ground for a fully automated takeoff performance monitor and control system. As well, given the high dynamics that such a vehicle would exhibit, inertial aiding of GPS tracking loops will be essential.
If the development of UCAV technology progresses as expected, the spinoffs will benefit commercial aviation. Once proven in an unmanned vehicle, the adoption of takeoff performance monitoring technology in passenger aircraft will be able to proceed with much less skepticism than is currently attached to such devices.

Figure 8.3 Boeing X-45A UCAV
REFERENCES


APPENDIX I

LIMITED SUPPLEMENTAL TYPE CERTIFICATE APPLICATION
APPENDIX II

TAKEOFFS FROM RUNWAY 15, SASKATOON AIRPORT
## APPENDIX III

### CD-ROM

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